Decreasing marginal guilt and psychological contract

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Abstract

We study a model of guilt aversion to explain other-regarding behavior which responses discontinuously to parameters. Existing theoretical research assumes that a decision maker feels guilt of which marginal value is increasing or constant. In contrast, our model assumes that the magnitude of an agent's marginal guilt is decreasing. Our model can describe anomalies reported by existing experimental researches. We demonstrate an application of our model for describing psychological contract in a principal-agent relation.

Keywords: guilt aversion, psychological games, dictator games, efficiency wage

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1. Introduction

Experimental studies have found that people regard not only their own payoff, but also the others' payoffs. Such preferences are called social preferences or other-regarding preferences. Inequity aversion is one of the most successful models describing otherregarding preferences (Fehr and Schmidt, 1999). However, there are anomalies of it in experimental literature. For example, List (2007) conducts an experiment of a dictator game in which a dictator can choose to "take" money from the other passive player. The model of inequity aversion predicts that a subject who donates when she has no "take" option also donates the same amount with the "take" option. However, List's experimental result is summarized as that people behave less altruistically when the "take" option is provided than when no such option is. Our analysis will describe such a behavior as a response discontinuously to the lower bound of a choice set.

In this study, we will establish a new model of guilt aversion. Guilt is the discomfort experienced when a decision maker violates the other's expectation or social norm. The model of guilt aversion is also employed to explain experimental results (see Battigalli and Dufwenberg (forthcoming) for the literature). In existing theoretical papers (e.g., Jensen and Kozlovskaya, 2016), the marginal magnitude of guilt is assumed to be increasing or constant with respect to the difference between the real choice and the other's expected choice. In contrast, our model is the first to assume that it is decreasing. We show that this model implies discontinuous behaviors and can explain the List's result.

Our model implies that a decision maker responses discontinuously to parameters at some threshold. Discontinuous actions are important in economic situations. We demonstrate an application of our model for describing psychological contract in a principalagent relation. Under the assumption of decreasing marginal guilt, the equilibrium wage is rigid in some range of market parameters, and can jump to the minimum level at a threshold.

This paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the model. Section 4 demonstrates an application. Section 5 concludes. In this preceeding paper, all proofs are abbreviated.

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2. Model

2.1. An environment

We consider a simple two-person game called a dictator game. In this game, one player (called a proposer) is endowed with a pie of fixed size and decides how much of it to donate to the other passive player. Formally, let $\sigma \in \mathbb{R}_{++}$ denote the fixed size and $\ell \in \mathbb{R}$ denote the lower bound of the proposer's choice set. In other words, the proposer chooses $x \in [\ell, \sigma]$, and she receives monetary payoff $\sigma - x$ while the other passive player (called a receiver) receives x. We assume that $\ell < \sigma$, and ℓ can be negative.

This setting describes the anomaly documented by List (2007) as follows: A proposer chooses $x^* > 0$ when $\ell = 0$, whereas she chooses $x^* = \ell < 0$ when $\ell < 0$. One research question of this study is how the lower bound ℓ affects a proposer's choice in such a way.

2.2. Beliefs

Our study considers a second-order belief of the proposer. We refer to what the passive player believes that the proposer chooses as the passive player's *first-order belief*. Moreover, we refer to what the first-order belief the proposer believes that the passive player has as the proposer's *second-order belief*. Let r denote the proposer's second-order belief, and assume that $\ell \leq r \leq \sigma$.

2.3. Guilt aversion

If a proposer is selfish, she chooses $x^* = \ell$ to maximize $\sigma - x$. In contrast, our analysis assumes that the proposer pays attention to *guilt* defined by the following function.

Definition (guilt function). A function $g : \mathbb{R}^2 \to \mathbb{R}_+$ is a guilt function if g(x,r) is strictly decreasing (resp. increasing) in x (resp. r) whenever x < r, and g(x,r) = 0 whenever $x \ge r$.

In words, the proposer feels more guilt when her donation x is smaller than the receiver's expectation r. Formally, we assume that the proposer maximizes a utility function given by

$$U(x,r) = u(\sigma - x) - g(x,r).$$
(1)

In (1), the function $u : \mathbb{R} \to \mathbb{R}$ denotes her monetary utility and the guilt function $g : \mathbb{R}^2 \to \mathbb{R}$ means psychological loss.

2.4. Marginal guilt

Assuming that a function u and g are differentiable, we can define a marginal rate of substitution (MRS) as

$$MRS(x_1, x_2, r) \coloneqq \left| \frac{\partial g(x_2, r)}{\partial x_2} \middle/ u'(x_1) \right|$$
(2)

for any $(x_1, x_2, r) \in \mathbb{R}^3$. This denotes (the absolute value of) a slope of an indifference curve with fixed r. Whenever $x_2 < r$, the indifference curve is down-sloping because uand g are assumed to be increasing in x_1 and x_2 , respectively. The below figures depict the indifference curves.



Figure 1. Indifference curves

In our main analysis, we assume on the utility function (1) the following condition:

Definition (Increasing Marginal Rate of Substitution). A utility function (1) satisfies *increasing marginal rate of substitution (IMRS)* if the functions u, g are differentiable and the $MRS(x_1, x_2, r)$ is increasing in x_2 for any (x_1, x_2, r) satisfying $x_2 < r$.

The IMRS assumption implies that a "marginal guilt" is decreasing. To explain this, we will consider x_2 satisfying $x_2 < r$ for fixed r. An intuition of the guilt function $g(\cdot, r)$ is that a decision maker feels more guilty as $r - x_2$ increases (i.e., x_2 decreases). Thus, a value of the guilt function is increasing in the difference $r - x_2$, and its marginal value measured by $u(x_1)$ is decreasing in $r - x_2$ when IMRS is satisfied. Therefore, we can interpret the IMRS as "decreasing marginal guilt."

2.5. Equilibrium

For the function $U: \mathbb{R}^2 \to \mathbb{R}$ given by (1), we define an equilibrium concept as follows:

Definition (Psychological Nash Equilibrium). A strategy-belief pair (x^*, r^*) is a *psychological Nash equilibrium* (PNE) if it satisfies

$$x^* \in \arg\max_{x \in [\ell,\sigma]} U(x^*, r^*), \tag{3}$$

$$x^* = r^*. \tag{4}$$

In words, (3) means that the proposer exhibits guilt aversion, and (4) means that the both players have "right" beliefs in a sense that the proposer's second-order belief r^* is identical to the passive player's first-order belief z^* , and the belief z^* is identical to the proposer's actual choice x^* .

3. Result

3.1. Benchmark case

Before the main case with the IMRS assumption, we here consider cases that the MRS is increasing or constant. For the two cases, we have shown the following facts.

Proposition 1. Let u be a differentiable function satisfying that $u''(x_1) < 0 < u'(x_1)$ for all $x_1 \in \mathbb{R}$. Then,

(a) If $MRS(\sigma - x, x, r) < 1$ is constant for any x, r such that x < r, then any strategy consists a PNE.

- (b) If $MRS(\sigma x, x, r) \ge 1$ is constant for any x, r such that $x < r, (\ell, \ell)$ is the unique PNE.
- (c) If $MRS(\sigma x, x, r)$ is increasing for any x, r such that x < r, there exists a unique PNE strategy x^* which is continuous in ℓ .

Cases (b) and (c) are obviously inconsistent with List's experimental result. Moreover, case (a) cannot explain why a significantly more amount of subjects chooses a negative offer when $\ell < 0$ than when $\ell = 0$.

3.2. Main case

Our main result is summarized by the below figure. In the diagram, suppose that the blue (thin) and green (thick) curves depict difference curves, and that the red line depicts the choice set. Then, observe that the decision maker chooses x = r if $\ell = 0$, whereas she chooses $x = \ell$ if $\ell = \ell^* < 0$. The blue (thin) curve indicates a utility level from offering x = r, while the green (thick) indicates one from offering $x = \ell$. The diagram suggests that the former U(r,r) is larger if $\ell^* < \ell$, whereas the later $U(\ell,r)$ is larger if $\ell < \ell^*$. The later case means r cannot be an equilibrium belief. Moreover, it suggests, under the IMRS assumption, that there exists such an $\ell^*(r)$ for given r. For any $\ell < \ell^*(\sigma)$, hence, the proposer must offer $x = \ell$ from $[\ell, \sigma]$ even if she offers x > 0 from $[0, \sigma]$.



Figure 2. Indifference curves with IMRS assumption

The intuition of Figure 2 is summarized by the following lemma.

Lemma 2. Let a utility function (1) satisfy IMRS. Then, for any $\sigma \in \mathbb{R}$,

- (i) the lowest strategy ℓ always a PNE strategy,
- (ii) any $r \in (r', \sigma]$ is a PNE strategy if $r' \in (\ell, \sigma]$ is another PNE strategy,
- (iii) any $r \in [\ell, \sigma]$ is a PNE strategy if and only if

$$MRS(\sigma - \ell, \ell, \sigma) \ge 1, \tag{5}$$

(iv) the lowest strategy ℓ is a unique PNE strategy if and only if (5) does not hold and

$$u(\sigma - \ell) > g(\ell, \sigma).$$
(6)

Because we assume IMRS, (5) holds for a sufficiently small $\ell \in \mathbb{R}$. That is, any proposer chooses the negative ℓ from a set $[\ell, \sigma]$ even if she choose a positive offer from $[0, \sigma]$. Hence, our model can describe List's (2007) experimental result. This is summarized as follows:

Proposition 3. Let a utility function (1) satisfy IMRS. Take any $\sigma \in \mathbb{R}$ satisfying

$$\lim_{\ell \to -\infty} MRS(\sigma - \ell, \ell, \sigma) < 1, \tag{7}$$

$$\lim_{z \to \infty} u(\sigma + z) > \lim_{z \to \infty} g(-z, \sigma).$$
(8)

Then, there exists $L \leq \sigma$ such that (ℓ, ℓ) is a unique PNE for any $\ell < L$.

Similarly, the condition (5) holds for a sufficiently large $\sigma \in \mathbb{R}$. This suggests that more people donate nothing in the real society with a higher σ than in laboratories with lower σ . This result is summaried as follows:

Proposition 4. Let a utility function (1) satisfy IMRS. Suppose that $\ell \in \mathbb{R}$ satisfies

$$\lim_{\sigma \to \infty} MRS(\sigma - \ell, \ell, \sigma) < 1, \tag{9}$$

$$\lim_{z \to \infty} u(z - \ell) > \lim_{z \to \infty} g(\ell, z).$$
(10)

Then, there exists $S \ge \ell$ such that (ℓ, ℓ) is a unique PNE for any $\sigma > S$.

These results suggest that the IMRS condition causes discountinuous behavior, and it can explain choice-set anomalies as List reported. We note that the above expectation does not require condition (4).

4. Application: psychological contract

4.1. Psychological contract

We apply the above result to wage theory. Consider a situation in which a firm sets a worker's wage, and after observing that, the worker chooses his effort level. We assume that there are no uncertainty and no information asymmetry, and that the firm cannot vary wages or anything according to the effort level. If the worker has no other-regarding preference, he has no incentive to make any effort. However, if he is guilt averse and can guess the principal's belief about his effort from the wage offer, he may make some effort. This is one of psychological contract, defined as a perception of an exchange agreement between oneself and another (Rousseau, 1989).

4.2. Setup

Consider the following two-stage model: First, a firm chooses a level of wage $w \in \mathbb{R}_+$. Second, a worker chooses effort level denoted by $e \in \mathbb{R}_+$. In this situation, there are no uncertainty and no information asymmetry, and the firm cannot vary wages according to the effort level. We suppose that the firm maximizes its profit given by

$$pf(e) - w, (11)$$

where its production function $f : \mathbb{R}_+ \to \mathbb{R}$ satisfies f''(e) < 0 < f'(e) for all $e \in \mathbb{R}_+$. The parameter $p \in \mathbb{R}_{++}$ is interpreted as the productivity, demand, or price of the output good. Also, we suppose that the worker maximizes

$$U(w, e) = w - c(e) - g(e, r),$$
(12)

where the worker's cost $c : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies c'(e), c''(e) > 0 for all $e \in \mathbb{R}_+$. We assume that r depends on offered w. Formally, let a differentiable function $\mu : \mathbb{R}_+ \to \mathbb{R}_+$ exist such that $r = \mu(w)$ and $\mu'(w) > 0$. In words, the firm and the worker agree a perception that the firm offers a higher wage when it believes that the worker will make higher efforts. In our study, we assume that g in (12) satisfies the IMRS condition and g'(0) > c'(0).

4.3. Implications

For given market parameter p, we solve the model by backward induction in which the worker chooses PNE strategy $e_p^*(w)$ for given wage $w \in \mathbb{R}_+$ and the firm chooses w on such an $e_p^*(\cdot)$. Note that each equilibrium $(w_p^*, e_p^*(\cdot), r^*)$ depends on the parameter p. As a benchmark case, if $g(\cdot)$ is always zero, then $e_p^*(w) = w_p^* = 0$ in the equilibrium. As the main case, suppose that the worker's payoff function (12) satisfies the IMRS condition. Then, the results are summarized as follows:

- (i) There is a range of p such that the equilibrium wage w_p^* is increasing in p.
- (ii) There is a threshold p' such that the equilibrium wage w_p^* is constant for any p > p'.
- (iii) There is a threshold p'' such that the equilibrium wage w_p^* is zero for any p < p''.

We interpret the results (i) and (ii) as efficiency wage and sticky wage, respectively. Result (iii) suggests that firms may have incentives to make sudden changes in wages or employment if it uses efficiency wages to make guilt averse workers make more efforts.

5. Conclusion

This paper studied a model of decreasing marginal guilt. Under the model, we show that a decision maker responses discontinuously to choice sets. This result is consistent with experimental studies such as List (2007). Moreover, we demonstrate an application of our model for describing psychological contract in a principal-agent relation. In the application, we show that our model implies that the equilibrium wage is rigid in some range of market parameters, and can jump to the minimum level at a threshold.

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