Comparison of Max–Min Expected Utility and Expectations-Based Reference-Dependent Preference in Static and Dynamic Portfolio Choice*

Yuki Shigeta†

September 27, 2022

Abstract

This study undertakes a comparison of max–min expected utility and expectations-based reference-dependent preference in portfolio choice. I use popular settings of both preferences. I show that these two preferences are observationally equivalent for static portfolio choice. Furthermore, the cross-sectional equilibrium expected return has the same representation as the capital asset pricing model. For dynamic portfolio choice, equivalence in optimal portfolio weight for both preferences holds only in a restricted parameter space. The optimal consumption–wealth ratio differs for each preference. Numerical comparative statics show that this partial equivalence result occurs when deviation of both preferences from the standard expected utility is small.

Key words: Max–Min Expected Utility, Expectations-Based Reference-Dependent Preference, Model Uncertainty, Loss Aversion, Portfolio Choice

JEL Classification: D81, G11, G12

1 Introduction

It is not uncommon for models in finance and economics to include an agent whose preference deviates from a neoclassical one. One of these deviations is loss aversion, which is one of key features of the prospect theory of Kahneman and Tversky (1979). However, the classical prospect theory assumes a given reference point that determines gain or loss. One solution to this exogeneity problem about a reference point is expectations-based reference-dependent preference (EBRDP), introduced by the seminal works of Kőszegi and Rabin (2006, 2007, 2009).An agent who has EBRDP chooses both options and beliefs regarding a reference point. Applications of the EBRDP, such as De Giorgi and Post (2011), Xie et al. (2018), and Pagel (2016, 2018), suggest that an agent who has this preference tends to be a less aggressive investor than the standard agent is.

Another deviation is probability distortion. Model uncertainty is particularly recognized as one of successful applications of probability distortion. Specifically, Gilboa and Schmeidler (1989) incorporate model uncertainty to

*I thank participants of the 28th annual meeting of Nippon Finance Association, and Masataka Suzuki (as a discussant of the meeting) for valuable comments. This is a short version of the original paper for the proceedings in the 16th annual meetings of Association of Behavioral Economics and Finance. This work was supported by JSPS KAKENHI Grant Numbers 17K03876 and 18K12811. All remaining errors are my own.

†Faculty of Economics, Tokyo Keizai University, Japan. E-mail address: sy46744@gmail.com

decision theory, and propose the max–min expected utility (MMEU) model in which an agent maximizes the worst-case expected utility. Applications of the MMEU model in the finance literature, such as Chen and Epstein (2002) and Garlappi et al. (2007), show that an MMEU agent tends to be a less aggressive investor than the standard agent.

One can naturally hypothesize equivalence on the MMEU and EBRDP from the above similarity. In this study, I analyze the equivalence and difference of these two preferences in a context of portfolio choice. I consider modifications of familiar applications of these two preferences: the max–min type of model uncertainty by Garlappi et al. (2007), and the EBRDP by Pagel (2016, 2018). When specifying the constant relative risk aversion (CRRA) consumption utility and normally distributed risky asset's return, I show that these two preferences are observationally equivalent on static portfolio choice if preference parameters are chosen appropriately. This equivalence also holds for the existence of multiple risky assets, even though this result is omitted in this short version due to page limitations.

In addition, I explore equivalence and difference of the two preferences on dynamic consumption–investment decision, assuming the log consumption utility. In this case, the optimal consumption is determined differently in the MMEU and EBRDP. The MMEU's optimal consumption–wealth ratio is a constant, and is the same as that of the standard expected utility, whereas the EBRDP's optimal consumption–wealth ratio depends on a current risky asset's return. Furthermore, the two preferences' optimal portfolio weights to a risky asset are equivalent only in some regions of preference parameters, even if these parameters are tuned. Therefore, the equivalence of the two preferences on dynamic decision is limited.

The novel contribution of this study to the literature is to analyze equivalence and difference on probability distortion and loss aversion on dynamic decisions. Other researchers also study equivalence on probability distortion and loss aversion on a static decision. Masatlioglu and Raymond (2016) and Ai et al. (2018) show that EBRDP is equivalent to rank-dependent utility, which involves distorting probabilistic belief, on a static decision. Lleras et al. (2019) show the equivalence between MMEU and a preference that reveals loss aversion in a one-period decision, using the decision-theoretic axiomatic approach. The abovementioned studies mainly focus on a one-period decision by allowing a large class of utility functions, but the present study also considers equivalence and difference on dynamic decisions, even though I specify the functional form of the consumption utility function to derive an explicit solution.

The reminder of this paper is organized as follows. Section 2 shows equivalence on static portfolio choice in the case of a single risky asset. Section 3 compares the dynamic decisions of the two preferences and conducts numerical comparative statics.

2 Single Risky Asset Case in Static Portfolio Choice

I begin with a single risky asset case. Suppose there are two assets: a risk-free asset and risky asset. The gross risk-free rate is denoted by R^f, which is a constant. The gross risky asset return is denoted by R, whose distribution is log-normal: $r := \log R \sim N(\mu - \sigma^2/2, \sigma^2)$, where μ and σ are constants. An investor chooses a portfolio weight of the risky asset, denoted by α. The investor's consumption C depends on R^f, R, α, and his or her initial wealth W such that $C = W(R + \alpha(R - R^f))$. This budget constraint can be approximated as follows:

$$C \approx W \exp\left\{ r^f + \alpha\left(r + \frac{\sigma^2}{2} - r^f\right) - \frac{\alpha^2\sigma^2}{2} \right\}, \tag{2.1}$$

where $r^f = \log R^f$.In continuous time, the wealth dynamics are represented as a stochastic differential equation: $\mathrm{d}W = W(r^f \mathrm{d}t + \alpha(\mathrm{d}S/S - r^f \mathrm{d}t))$, where S is a risky asset price. If $\mathrm{d}S/S = \mu \mathrm{d}t + \sigma \mathrm{d}B$, where B is a standard Brownian motion, then a log return of S is $r := \log S_1 - \log S_0 = \mu - \sigma^2/2 + \sigma B_1 \sim N(\mu - \sigma^2/2, \sigma^2)$. In addition, if α is fixed, then $W_1 = W_0 \exp\{r^f + \alpha(\mu + \sigma B_1 - r^f) - \alpha^2\sigma^2/2\} = W_0 \exp\{r^f + \alpha(r + \sigma^2/2 - r^f) - \alpha^2\sigma^2/2\}$. Thus, the approximation (2.1) coincides with the case of $C = W_1$.

I now consider an MMEU investor. The MMEU investor has doubt about the correctness of the risky asset's expected return that he considers ex ante. His utility function is

$$U := \min_{\psi} \mathrm{E}^{\psi}[u(C)], \quad \text{subject to } \frac{\psi^2}{\sigma^2} \leq \epsilon^2, \tag{2.2}$$

where u is a felicity function and ψ is represented as model uncertainty about the risky asset's expected return: under a probability measure induced by the expectation operator E^{ψ}, the distribution of r is $N(\mu+\psi-\sigma^2/2, \sigma^2)$. ϵ is a non-negative constant, which is expressed as the degree of model uncertainty that the investor is exposed to. A large ϵ allows ψ to take a large positive or negative value, so that the MMEU investor greatly doubts the credibility of the value of μ. The MMEU investor solves the minimization problem in (2.2) subject to the constraint.The minimizer of $\mathrm{E}^{\psi}[u(C)]$ depends on the portfolio weight α. In turn, the MMEU investor chooses α to maximize U: $\max_{\alpha} U = \max_{\alpha} \min_{\psi} \mathrm{E}^{\psi}[u(C)]$. I assume that the felicity function is CRRA utility, $u(c) = c^{1-\gamma}/(1-\gamma)$, where $\gamma \neq 1$ is a positive constant, which is expressed as a coefficient of relative risk aversion.

By approximation (2.1), the expectation, $\mathrm{E}^{\psi}[u(C)]$, can be approximated to

$$\mathrm{E}^{\psi}[u(C)] \approx \frac{W^{1-\gamma}}{1-\gamma} \exp\left\{(1-\gamma)\left(r^f + \alpha(\mu+\psi-r^f) - \gamma\frac{\alpha^2\sigma^2}{2}\right)\right\}$$

Hence, the max–min problem $\max_{\alpha} \min_{\psi} \mathrm{E}^{\psi}[u(C)]$ can be transformed to

$$\max_{\alpha} \min_{\psi} \left\{r^f + \alpha\left(\mu+\psi-r^f\right) - \gamma\frac{\alpha^2\sigma^2}{2}\right\}, \tag{2.3}$$

subject to $\psi^2/\sigma^2 \leq \epsilon^2$ According to Garlappi et al. (2007), when $\alpha \neq 0$, the inner minimization problem can be solved as follows:

$$\min_{\psi} \left\{r^f + \alpha\left(\mu+\psi-r^f\right) - \gamma\frac{\alpha^2\sigma^2}{2}\right\} = r^f + \alpha\left(\mu-r^f\right) - \gamma\frac{\alpha^2\sigma^2}{2} - \epsilon|\alpha|\sigma, \tag{2.4}$$

and the minimizer is $\psi^* = -\epsilon\sigma\alpha/|\alpha|$. When $\alpha = 0$, the inner minimization problem has multiple solutions: the minimizer ψ^* is not determinate. However, (2.4) then takes the same value as the original objective function, (2.3). Therefore, (2.4) includes the case in which $\alpha = 0$.

Applying the standard optimization method without constraints to the maximization of (2.4), I obtain the MMEU investor's optimal portfolio. This proof including the multiple risky assets case can be found in Shigeta (2017).

Proposition 1 *The MMEU investor's optimal portfolio weight* α^* *is*

$$\alpha^* = \frac{1}{\gamma}\left(1 - \frac{\epsilon}{|SR|}\right)^+ \frac{\mu - r^f}{\sigma^2}, \tag{2.5}$$

where $(x)^+ = \max\{x, 0\}$ *and* $SR = \dfrac{\mu - r^f}{\sigma}$.

Proposition 1 implies that if the degree of model uncertainty ϵ is larger than the absolute value of the ex ante Sharpe ratio, $|SR|$, then the MMEU investor does not hold the risky asset. This behavior makes sense, because the risk-free asset is not exposed to any model uncertainty.

Here, I turn to the case of the EBRDP investor. I follow the setting of Pagel (2018). Comparing the MMEU investor's optimal portfolio weight (2.5) to the optimal portfolio weight of Pagel (2018)'s EBRDP investor in static choice, equivalence on the two optimal portfolio weights can be inferred. However, it is not certain what causes this equivalence and when the equivalence occurs, because Pagel (2018) does not derive an explicitly analytical representation of the objective function. I now show that this equivalence is rooted in the equivalence of their objective functions. The EBRDP investor's utility is $U := \mathrm{E}\left[u(C) + n(C, F_C)\right]$, where u is a felicity function and n is news utility. News utility is defined as

$$n(C, F_C) := \eta\left(\int_0^C \Big(u(C) - u(c)\Big)\mathrm{d}F_C(c) + \lambda\int_C^\infty \Big(u(C) - u(c)\Big)\mathrm{d}F_C(c)\right),$$

where F_C is the cumulative distribution function of consumption C, and $\eta > 0$ and $\lambda > 1$ are constants that represent the weight of news utility and the degree of gain–loss asymmetry, respectively. News utility is outcome-wise reference-dependent, such that $u(C) - u(c)$ is multiplied by η if $u(C) - u(c) > 0$ and it is multiplied by $\lambda\eta$ otherwise. Since

$\lambda > 1$, bad news, such as $C < c$, is more weighted toward good news $C > c$. As well as the MMEU case, the felicity function is a CRRA function, $u(c) = c^{1-\gamma}/(1-\gamma)$. After simple calculation, the expected value of news utility is transformed to

$$\mathrm{E}\left[n(C, F_C)\right] = \mathrm{E}\left[\eta(\lambda - 1)\int_C^\infty \Big(u(C) - u(c)\Big)\mathrm{d}F_C(c)\right] = \mathrm{E}\left[\eta(\lambda-1)\Big(\frac{C^{1-\gamma}}{1-\gamma} - \frac{\widetilde{C}^{1-\gamma}}{1-\gamma}\Big)\mathbb{1}\{0 \geq C - \widetilde{C}\}\right],$$

where $\mathbb{1}\{\cdots\}$ is an indicator function that takes one if $\{\cdots\}$ is true and zero otherwise, and $\widetilde{C}$ is an independent copy of C, i.e., an auxiliary random variable whose distribution is the same as that of C, but it is independent of C. By the definition of $\widetilde{C}$ and approximation (2.1), $\widetilde{C}$ can be approximated to $\widetilde{C} \approx W\exp\{r^f + \alpha\left(\widetilde{r} + \sigma^2/2 - r^f\right) - \alpha^2\sigma^2/2\}$, where $\widetilde{r}$ is an independent copy of the return r: a random variable whose distribution is the same as that of r, but is independent of r. Furthermore, I use a linear approximation of the exponential function: $e^x \approx 1 + x$. Then, the expected value of the news utility can be approximated to $\mathrm{E}\left[n(C, F_C)\right] \approx W^{1-\gamma}\,\mathrm{E}\left[\eta(\lambda-1)\alpha(r - \widetilde{r})\mathbb{1}\{0 \geq \alpha(r - \widetilde{r})\}\right]$. The consumption expected utility $\mathrm{E}[u(C)]$ can be also approximated to $\mathrm{E}[u(C)] \approx u(W)\exp\{(1-\gamma)(r^f + \alpha(\mu - r^f) - \gamma\alpha^2\sigma^2/2)\} \approx W^{1-\gamma}(1/(1-\gamma) + r^f + \alpha(\mu - r^f) - \gamma\alpha^2\sigma^2/2)$. Finally, the EBRDP investor's optimization problem can be reduced to the following problem:

$$\max_\alpha \left\{r^f + \alpha\left(\mu - r^f\right) - \gamma\frac{\alpha^2\sigma^2}{2} + \mathrm{E}\left[\eta(\lambda-1)\alpha(r - \widetilde{r})\mathbb{1}\{0 \geq \alpha(r - \widetilde{r})\}\right]\right\}.$$

Then the following proposition holds.

Proposition 2 *The EBRDP investor's optimal portfolio weight is the same as the MMEU investor's optimal portfolio weight if* $\epsilon = \eta(\lambda - 1)/\sqrt{\pi}$.

Intuitively, the MMEU investor's model uncertainty works as a model-misspecification error worsens his utility. The larger the position of the risky asset that the MMEU investor has, the greater model uncertainty he is exposed to with respect to the risky asset. That leads the MMEU investor to decrease his risky asset position, and thus, the absolute value of his risky asset portfolio multiplied by a negative constant occurs in his utility. Meanwhile, the EBRDP investor's loss aversion via news utility works as follows: a bad investment result weights her utility greater than the standard utility. The larger the position of the risky asset that the EBRDP investor has, the greater is her exposure to a possible negative impact when an investment result is bad. That leads the EBRDP investor to decrease her risky asset position similarly to the MMEU investor, and thus, the absolute value of her risky asset portfolio is multiplied by a negative constant in her utility.

3 Dynamic Consumption–Investment Decision

I now explore equivalence and difference between MMEU and EBRDP for dynamic consumption–investment decisions. For simplicity, I consider the case of one risk-free asset and one risky asset. As well as the static case, the risk-free rate is constant, $r^f = \log R^f$, and the gross risky asset return R_t for each time t follows the log-normal distribution: $r_t = \log R_t \sim N(\mu - \sigma^2/2, \sigma^2)$, where μ and σ are constants. Furthermore, I consider an independent and identically distributed (i.i.d.) market: for any i and j, R_i and R_j are mutually independent.

At any time, the investor consumes a fraction of current wealth and invests the rest in the risk-free asset and risky asset. Thus, the investor's budget constraint at time t is as follows.

$$W_{t+1} = (W_t - C_t)\Big(R^f + \alpha_t(R_{t+1} - R^f)\Big), \tag{3.1}$$

where C_t is time-t consumption and α_t is a portfolio at time t. W_t is wealth at time t. I denote the dynamic equation (3.1) by $W_{t+1} = \mathcal{F}(r_{t+1}, W_t, \alpha_t, C_t)$. Thanks to the log-linearization, the budget constraint (3.1) can be approximated to

$$\log W_{t+1} \approx \log(W_t - C_t) + r^f + \alpha_t\Big(r_{t+1} + \frac{\sigma^2}{2} - r^f\Big) - \frac{\alpha_t^2\sigma^2}{2}. \tag{3.2}$$

The investor obtains the utility from consumption at each time, and his or her felicity function is, unlike the static case, log-utility: $u(c) = \log c$. Furthermore, I consider the infinite horizon problems to obtain an explicit result.

First, I consider the MMEU investor's dynamic decision. The dynamic MMEU investor's problem is naturally extended from the static problem. Let $\mathrm{E}_t^{\{\psi_{t+\tau}\}}$ be a conditional expectation operator at time t under which $r_{t+\tau} \sim N(\mu + \psi_{t+\tau} - \sigma^2/2, \sigma^2)$ for all $\tau \geq 1$. Let β be the constant rate of time preference on $(0,1)$. Then, the MMEU investor's utility maximization problem at time t is

$$\max_{\{C_{t+\tau},\alpha_{t+\tau}\}_{\tau=0,1,2,\ldots}} \left\{ \log C_t + \min_{\{\psi_{t+\tau}\}_{\tau=1,2,\ldots}} \mathrm{E}_t^{\{\psi_{t+\tau}\}} \left[\sum_{\tau=1}^{\infty} \beta^\tau \log C_{t+\tau} \right] \right\},$$

subject to (3.2) and $\psi_{t+\tau}^2/\sigma^2 \leq \epsilon^2$ for $\tau = 1, 2, \cdots$. $\epsilon \geq 0$ is the (constant) degree of model uncertainty that the MMEU investor is exposed to.

Proposition 3 *The MMEU investor's optimal consumption–wealth ratio and portfolio are*

$$\frac{C_t^{MMEU*}}{W_t} = \rho^{MMEU*} = 1 - \beta, \quad \textit{and} \quad \alpha_t^{MMEU*} = \left(1 - \frac{\epsilon}{|SR|}\right)^+ \frac{SR}{\sigma}, \tag{3.3}$$

where SR is the one-period Sharpe ratio, $SR := (\mu - r^f)/\sigma$.

Proposition 3 shows that the MMEU investor's optimal consumption choice is the same as the case of the standard expected utility. This result depends on the choice of the felicity function. In this case, the felicity function is a natural logarithm. The log felicity function leads to the income effect completely canceling out the intertemporal substitution effect, which means that consumption choice is independent of the expected investment performance that is involved in model uncertainty. Meanwhile, the MMEU investor's optimal portfolio for dynamic choice is the same as that for the MMEU investor's static choice. Therefore, the model uncertainty affects only the MMEU investor's portfolio choice, not his consumption choice, if his felicity function is log utility.

Here, I consider the EBRDP investor's problem. The EBRDP investor's dynamic lifetime utility at time t is as follows:

$$U_t = \log C_t + n(C_t, F_{C_t}^{t-1}) + \mathrm{E}_t \left[\sum_{j=1}^{\infty} \beta^j \left(\log(C_{t+j}) + n(C_{t+j}, F_{C_{t+j}}^{t+j-1}) \right) \right], \tag{3.4}$$

where n is a (contemporaneous) news utility function as defined in the static problem. The utility function (3.4) is introduced by Kőszegi and Rabin (2006, 2009) and extended by Pagel (2016). In the monotone-personal equilibrium, at each time t, the EBRDP investor optimally chooses her policies as a monotone increasing function of the realized return r_t, and this optimal policy is consistent with the prospect $\mathcal{G}_c$. Therefore, we have the following definition of the monotone-personal equilibrium in this study.

Definition 4 (Monotone-Personal Equilibrium) *Let $\mathcal{G}(r,w) = (\mathcal{G}_a(r,w), \mathcal{G}_c(r,w)) \in \mathbb{R} \times (0,\infty)$ be a measurable function of the realized return r and the current wealth w. Then, $\{\alpha_t^{EBRDP*}, C_t^{EBRDP*}\}_{t=1,2,\ldots}$ is a* monotone-personal equilibrium *if it satisfies the following: (1) it maximizes the EBRDP investor's utility for a given $\{\widetilde{C}_t\}_{t=1,2,\ldots} = \{\mathcal{G}_c(\widetilde{r}_t, \mathcal{F}(W_{t-1}^{0,w;(\alpha,C)}, \widetilde{r}_t, \alpha_{t-1}, C_{t-1}))\}_{t=1,2,\ldots}$ as future hypothetical prospects, where $\widetilde{r}_t$ is an independent copy of r_t; (2) it holds that $(\alpha_t^{EBRDP*}, C_t^{EBRDP*}) = \mathcal{G}(r_t, W_t^{0,w;(\alpha^{EBRDP*},C^{EBRDP*})})$ for each time t; and (3) $r \to \mathcal{G}_c(r, \mathcal{F}(w', r, a, c))$ is monotone increasing and differentiable for any $(w', a, c) \in (0,\infty) \times \mathbb{R} \times (0,\infty)$.*

Based on Definition 4, this study's model has a monotone-personal equilibrium as follows:

Proposition 5 *In a monotone-personal equilibrium, the EBRDP investor's optimal consumption–wealth ratio is*

$$\frac{C_t^{EBRDP*}}{W_t} = \rho^{EBRDP*}(r_t) = \frac{1}{1 + \frac{\beta}{1-\beta} \frac{1}{1 + \eta F_r(r_t) + \eta\lambda(1 - F_r(r_t))}}, \tag{3.5}$$

where F_r is the cumulative distribution function of r_t. The EBRDP investor's optimal portfolio weight cannot be expressed in a closed-form. However, it is independent of the return realization and can be approximated to

$$\alpha_t^{EBRDP*} \approx \left(1 - (1-\beta)\frac{\eta(\lambda-1)}{\sqrt{\pi}|SR-k|}\right)^+ \frac{SR-k}{\sigma} + \frac{k}{\sigma}, \tag{3.6}$$

where k is a positive constant that depends only on preference parameters η, λ, and β.

If λ is sufficiently small, then the approximated optimal portfolio (3.5) coincides with the MMEU optimal portfolio when $\epsilon = (1-\beta)\eta(\lambda-1)/\sqrt{\pi}$.

References

Ai, J., L. Zhao, and W. Zhu. 2018. Portfolio Choice in Personal Equilibrium. *Economics Letters* 170:163–167. URL https://doi.org/10.1016/j.econlet.2018.06.018.

Chen, Z., and L. G. Epstein. 2002. Ambiguity, Risk, and Asset Returns in Continuous Time. *Econometrica* 70:1403–1443. URL https://doi.org/10.1111/1468-0262.00337.

De Giorgi, E. G., and T. Post. 2011. Loss Aversion with a State-Dependent Reference Point. *Management Science* 57:1094–1110. URL https://doi.org/10.1287/mnsc.1110.1338.

Garlappi, L., R. Uppal, and T. Wang. 2007. Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach. *The Review of Financial Studies* 20:41–81. URL https://doi.org/10.1093/rfs/hhl003.

Gilboa, I., and D. Schmeidler. 1989. Maxmin Expected Utility with Non-Unique Prior. *Journal of Mathematical Economics* 18:141–153. URL https://doi.org/10.1016/0304-4068(89)90018-9.

Kahneman, D., and A. Tversky. 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica* 47:263–291. URL https://doi.org/10.2307/1914185.

Kőszegi, B., and M. Rabin. 2006. A Model of Reference-Dependent Preferences. *The Quarterly Journal of Economics* 121:1133–1165. URL https://doi.org/10.1093/qje/121.4.1133.

Kőszegi, B., and M. Rabin. 2007. Reference-Dependent Risk Attitudes. *American Economic Review* 97:1047–1073. URL https://doi.org/10.1257/aer.97.4.1047.

Kőszegi, B., and M. Rabin. 2009. Reference-Dependent Consumption Plans. *American Economic Review* 99:909–936. URL https://doi.org/10.1257/aer.99.3.909.

Lleras, J. S., E. Piermont, and R. Svoboda. 2019. Asymmetric Gain–Loss Reference Dependence and Attitudes toward Uncertainty. *Economic Theory* 68:669–699. URL https://doi.org/10.1007/s00199-018-1138-4.

Masatlioglu, Y., and C. Raymond. 2016. A Behavioral Analysis of Stochastic Reference Dependence. *American Economic Review* 106:2760–2782. URL https://doi.org/10.1257/aer.20140973.

Pagel, M. 2016. Expectations-Based Reference-Dependent Preferences and Asset Pricing. *Journal of the European Economic Association* 14:468–514. URL https://doi.org/10.1111/jeea.12137.

Pagel, M. 2018. A News-Utility Theory for Inattention and Delegation in Portfolio Choice. *Econometrica* 86:491–522. URL https://doi.org/10.3982/ECTA14417.

Shigeta, Y. 2017. Portfolio Selections under Mean-Variance Preference with Multiple Priors for Means and Variances. *Annals of Finance* 13:97–124. URL https://doi.org/10.1007/s10436-016-0291-7.

Xie, Y., S. Hwang, and A. A. Pantelous. 2018. Loss Aversion around the World: Empirical Evidence from Pension Funds. *Journal of Banking and Finance* 88:52–62. URL https://doi.org/10.1016/j.jbankfin.2017.11.007.