Preference for randomization and validity of random incentive system under ambiguity: An experiment*

Tomohito Aoyama[a] Nobuyuki Hanaki[b]

September 18, 2021

Abstract

Random Incentive System (RIS) is a standard method to incentivize participants in economic experiments. However, recent theoretical studies point out a possibility of its failure under ambiguity. We propose a modification of RIS, named *independent RIS* (I-RIS), with the aim of improving its reliability. We conducted an experiment to evaluate the performances of the standard RIS and I-RIS in direct and indirect manners. As a result, a non-negligible fraction of participants are not consistent with the reversal-of-order axiom. However, randomization attitudes do not explain inconsistent choices under RIS. In addition, we did not find significant differences in the performances between the RIS and I-RIS. These results suggest that preferences for randomization, which is driven by non-neutral ambiguity attitudes, do not cause the concerned failure of RIS.

Keywords: Random Incentive System, Ambiguity, Incentive Compatibility

JEL Code: C91, D81

1 Introduction

The random incentive system (RIS) is a standard method to incentivize participants in economic experiments (Azrieli et al. 2018). However, recent theoretical studies point out a possibility of its failure when participants concern about ambiguity (Bade 2015, Kuzmics 2017, and Oechssler and Roomets 2014). They argue that participants may view the whole experiment as one choice problem and exploit randomness of RIS to hedge ambiguity. If their conjecture holds, it is difficult to experimentally investigate choice behavior under ambiguity.

We propose a variation of RIS, named *independent RIS (I-RIS)* in which choice alternatives in different choice situations suffer ambiguity from independent ambiguity sources. In addition, we posit a preference condition, which is a variation of compound independence axiom of Segal (1990), that guarantees the incentive compatibility of I-RIS. This preference condition is satisfied by, for example, the maxmin expected utility with independent multiple priors (Gilboa and Schmeidler, 1989).

To test the performance of the standard RIS and I-RIS empirically, we conducted an experiment in which we elicit participants' preferences under ambiguity and relate them to behavior under RIS. Our experiment consists of two parts, which are conducted two weeks apart. In the first part, participants face a series of lotteries and report willingness-to-pay (WTP), which allows us to elicit participants' attitude toward ambiguity, randomization, and reversal of uncertainty resolution order. In the second part, participants face two *identical* binary choice problems under ambiguity. If RIS is incentive compatible, participants should choose the same lottery twice in the second part. If a participant chooses different lotteries (we call such choices as inconsistent choices), instead, we conclude s/he does not report his or her preferences truthfully.

*We gratefully acknowledge financial support from Joint Usage/Research Center at ISER, Osaka University, and Japan Society for the Promotion of Science (18K19954, 20H05631). The experiment reported in this paper has been approved by IRB of ISER, Osaka University.

[a]Hitotsubashi Institute for Advanced Study, Hitotsubashi University. E-mail: t.aoyama@r.hit-u.ac.jp

[b]Institute of Social and Economic Research, Osaka University. Email: nobuyuki.hanaki@iser.osaka-u.ac.jp

2 Independent random incentive system

In this section, we propose a variant of RIS, in which choice alternatives in different choice situations depend on different state spaces. We provide a preference condition that guarantees the incentive compatibility of our incentive scheme.

2.1 Setup

First, we formally introduce choice alternatives that appear in experiments on ambiguity. For any topological space Y, let ΔY denote for the set of Borel probabilities over Y. Let X be the outcome space, which is a compact metrizable set. Let $S_1, \dots, S_K$ be finite state spaces and let $\Omega = \prod_{k=1}^{K} S_k$. Each S_k is interpreted as a different ambiguity source. An act is a mapping $f : \Omega \to \Delta X$ and we denote the set of all the acts as $\mathcal{F}$.[1] A random act $P \in \Delta\mathcal{F}$ is a probability over $\mathcal{F}$. Let $\succeq$ be a continuous weak order over $\Delta\mathcal{F}$, which is interpreted as preferences of a decision maker (DM, henceforth). A second-order random act $\mathcal{P} \in \Delta(\Delta\mathcal{F})$ is a probability over $\Delta\mathcal{F}$. Each $x \in X$ is identified with $f_x \in \mathcal{F}$ such that $f_x(\omega) = x$ for all ω. Each $f \in \mathcal{F}$ is identified with the degenerate probability $\delta_f \in \Delta F$. Each $P \in \Delta\mathcal{F}$ is identified with the degenerate probability $\delta_P \in \Delta(\Delta\mathcal{F})$.

2.2 Incentive scheme

Next, we develop a framework to describe experiments on ambiguity and incentive schemes following Azrieli et al. (2020). Let $\mathcal{D} = \{D_1, \dots, D_N\}$ be a list of choice situations in the experiment, where D_n is a compact subset of $\Delta\mathcal{F}$. That is, participants report his/her best-preferred alternatives from each D_l.

Under ambiguity, RIS may fail partly because choice alternatives in different choice situations depend on a common ambiguity source. In order to avoid this pitfall, we consider the choice situations in which this is not the case. For $I \subset \{1, \dots, K\}$, let $S_I = \prod_{k\in I} S_k$ and denote typical elements of S_I as s_I, s'_I, etc. Let $S_{-I} = \prod_{k\notin I} S_k$. We denote $\mathcal{F}_I = \{f \in \mathcal{F} | f(s_I, s_{-I}) = f(s_I, s'_{-I})$ for any $s_I \in S_I, s_{-I}, s'_{-I} \in S_{-I}\}$.

Definition 1. *$\mathcal{D} = \{D_1, \dots, D_N\}$ is independent if there exists a partition $\{I_1, \dots, I_N\}$ of $\{1, \dots, K\}$ and $D_n \subset \Delta\mathcal{F}_{I_n}$ for each n.*

Given $\mathcal{D}$, the DM reports her choices $m = (m_1, \dots, m_N) \in M := \prod_n D_n$. The mechanism we consider randomly pays one of $m_1, \dots, m_N$. If a participant views the whole experiment as a single decision problem, s/he would manipulate reports to obtain better outcomes. To capture such underlying preferences, assume $\succeq$ extends to a relation $\succeq^*$ over $\Delta(\Delta\mathcal{F})$, which is also a continuous weak order. Whereas $\succeq$ represents the *true preferences* the experimenter is concerned about, $\succeq^*$ dictates the *ovserved choices*.

Definition 2 (Random incentive system)**.** *An RIS is a mapping $\varphi : M \to \Delta(\Delta\mathcal{F})$ such that there exists a full-support probability $\lambda \in \Delta\mathcal{D}$ and*

$$\varphi(m)(P) = \sum_{n;\, m_n = P} \lambda(D_n) \tag{1}$$

holds for any $m \in M$ and $P \in \bigcup_{n=1}^{N} D_n$.

For D_n and $\succeq$, let $\arg\max_{\succeq} D_n = \{P \in D_n | P \succeq Q \text{ for all } Q \in D_n\}$. Define $\arg\max_{\succeq^*} \varphi(M) = \{\varphi(m) | \varphi(m) \succeq \varphi(m') \text{ for all } m' \in M\}$.[2] We say RIS is incentive compatible if the reported best-preferred items are actually the best-preferred ones in each choice situation. That is,

Definition 3 (Incentive compatibility)**.** *RIS is incentive compatible if $\varphi(m) \in \arg\max_{\succeq^*} \varphi(M)$ implies $m_n \in \arg\max_{\succeq} D_n$ for all n.*

2.3 Preference condition

Next, we propose a condition that restricts the relationship between $\succeq$ and $\succeq^*$. For second-order random acts $\mathcal{P}_1, \dots, \mathcal{P}_l$ and $\alpha_1, \dots, \alpha_L \geq 0$ with $\sum_{l=1}^{L} \alpha_l = 1$, define the mixture $\sum_{l=1}^{L} \alpha_l \mathcal{P}_l \in \Delta(\Delta\mathcal{F})$ by $(\sum_{l=1}^{L} \alpha_l \mathcal{P}_l)(G) = \sum_{l=1}^{L} \alpha_l \mathcal{P}_l(G)$ for any Borel sets $G \subset \Delta\mathcal{F}$.[3] This operation also applies to first-order random acts because they are identified with degenerate second-order random acts.

[1] The set $\mathcal{F}$ is endowed with the product topology.

[2] The continuity of $\succeq$ and $\succeq^*$, the compactness of each D_l, and the continuity of φ guarantees that these two sets are nonempty.

[3] For the case $L = 2$, we write $\sum_{l=1}^{2} \alpha_l \mathcal{P}_l$ also as $\alpha_1\mathcal{P}_1 + (1 - \alpha_1)\mathcal{P}_2$.

Condition 1. *For any $I, I' \subset \{1, \ldots, K\}$, $P, Q \in \Delta\mathcal{F}_I$, $R \in \Delta\mathcal{F}_{I'}$, and $\alpha \in (0, 1]$, if $I \cap I' = \emptyset$,*

$$P \succeq Q \Leftrightarrow \alpha P + (1-\alpha)R \succeq^* \alpha Q + (1-\alpha)R. \tag{2}$$

Condition 1 contains two preference relations, $\succeq$ and $\succeq^*$. As noted above, whereas $\succeq^*$ dictates the observed data, $\succeq$ is not directly observed. Thus, this condition is what to assume, rather than to test. It guarantees the incentive compatibility of RIS with independent choice situations.

Proposition 1. *Any RIS is incentive compatible if $\mathcal{D}$ is independent and $(\succeq, \succeq^*)$ satisfies Condition 1.*

We say an experiment uses *independent RIS* if its choice situations $\mathcal{D}$ are independent and its rewards are determined by an RIS.

3 Experiment

Our experiment is intended to test the validity of RIS in indirect and direct manners. First, we measure participants' attitudes toward ambiguity, randomization, and reversal of uncertainty resolution order. Second, we compare the performance of the standard RIS and I-RIS. Specifically, we ask the following:

- Are more randomization seeking participants more likely to misreport their preferences under the standard RIS?
- Does the use of I-RIS improves incentive compatibility compared to the standard RIS?
- Does the improvement by I-RIS, if any, depend on randomization attitude?

3.1 Part one – preference elicitation

3.1.1 Task and incentive

In this part, participants face a series of choice situations where they are asked to report their WTP for a lottery, which pays an uncertain amount of money. Here we use the Becker–DeGroot–Marschak (BDM) method (Becker et al., 1964) together with I-RIS to eilict WTPs. The lotteries are classified into the following four classes in terms of the sources of uncertainty.

- *Risky lotteries* that pay monetary rewards contingent on a roll of a four-sided fair die.
- *Ambiguous lotteries* that are obtained from risky lotteries by changing the source of uncertainty, from a die to a four-color Ellsberg urn, while keeping possible monetary outcomes the same.
- *Coin–ball lotteries* that resolve in two steps: first a fair coin is tossed and then a ball is drawn from the Ellsberg urn. From an ambiguous lottery, we constructed a corresponding coin–ball lottery in the following way. Suppose the ambiguous lottery f that gives f_1, f_2, f_3, and f_4 JPY when the drawn ball is red, blue, green, and yellow, respectively. Now, construct a corresponding symmetric lottery f' that gives f_4, f_3, f_2, and f_1 JPY when the ball is red, blue, green, and yellow, respectively. The coin–ball lottery corresponding to f reduces to f when the coin lands with heads up, but it reduces to f' when the coin lands with tails up.
- *ball–coin lotteries* that are obtained from coin–ball lotteries by reversing the order of a coin toss and a ball draw.

3.1.2 Preference indices

From WTPs that we have elicited, we construct three indices, *the ambiguity attitude index* (I_A), *the preference for randomization index* (I_{PR}), and *the reversal-of-order index* (I_{RO}), to summarize the preference characteristics of each participant. Suppose the participant's reported WTPs of risky and ambiguous lotteries are $V^R = (v_1^R, \ldots, v_{10}^R)$ and $V^A = (v_1^A, \ldots, v_{10}^A)$, where v_i^R is the WTP of the ith risky lottery and similarly for ambiguous lotteries. Then, the I_A of the participant is defined by

$$I_A = |\{i | v_i^R > v_i^A\}| - |\{i | v_i^R < v_i^A\}| \tag{3}$$

as a measure of his/her degree of ambiguity aversion. I_A ranges from -10 to 10. In the analyses below, we label participants as ambiguity averse or seeking when $I_A \geq 4$ or $I_A \leq -4$, respectively, and as ambiguity neutral when neither is the case.

Similarly, suppose the participant's WTP in coin–ball lotteries and ball–coin lotteries are V^{CB} and V^{BC}. Then, we define I_{PR} and I_{RO} as

$$I_{PR} = |\{i|v_i^{CB} > v_i^A\}| - |\{i|v_i^{CB} < v_i^A\}|, \tag{4}$$

$$I_{RO} = |\{i|v_i^{BC} > v_i^{CB}\}| - |\{i|v_i^{BC} < v_i^{CB}\}|. \tag{5}$$

To interpret I_{PR}, observe that the ith coin–ball lottery is a fair lottery over two ambiguous lotteries such that one of them is obtained from the other by interchanging ball colors. Moreover, one of such lotteries is the ith ambiguous lottery. Thus, assuming symmetric beliefs over the likelihood of ball colors, comparing corresponding ambiguous/coin–ball lotteries gives a degree of preference for randomization.

I_{RO} is calculated by comparing the valuations of ball–coin lotteries and coin–ball lotteries. We are interested in this index because it reflects participants' conformity to the reversal-of-order axiom (Anscombe and Aumann 1963, Seo 2009). If a participant is not ambiguity neutral, the reversal-of-order axiom implies his/her nonneutrality to randomization, which implies the failure of RIS follows (Azrieli et al. 2020, Bade 2015). Thus, we are interested in whether the participants' preferences satisfy the reversal-of-order axiom.

Hypothesis 1. *The participants' preferences satisfy the reversal-of-order axiom.*

If this hypothesis is true, his/her I_{RO} is zero because s/he is indifferent between the corresponding coin–ball lottery and ball–coin lottery, which are the same except the timing of uncertainty resolution: $v_i^{BC} - v_i^{CB} = 0$, $i = 1, \ldots, 10$. Thus Hypothesis 1 is rejected if these numbers are significantly different from zero.

3.2 Part two – Comparison of RIS and I-RIS

3.2.1 Task and incentive

In Part 2, participants are informed that they will face two *identical* two-alternative choices. One of the lotteries pays 510 JPY if the drawn ball is red or blue and 10 JPY if the drawn ball is green or yellow, whereas the other lottery pays 0 JPY if the drawn ball is red or blue and 500 JPY if the drawn ball is green or yellow. Because participants are instructed about the repetition of the choice situation, if an incentive scheme employed here is incentive compatible and a participant is not indifferent between these two lotteries, s/he should choose the same lottery twice.

There are two treatments, *Independent* (IND) and *Correlated* (COR), in which we used different incentive schemes. Whereas the standard RIS is used in COR, I-RIS is used in IND. In both treatments, one of two choice situations is randomly picked to use for compensation at the beginning of the experiment. The difference between the treatments is in the final stage of the experiment where the uncertainties of lotteries are resolved. Whereas only one ball is drawn in COR, four balls are independently drawn with replacement in IND; they are used to evaluate four lotteries that were presented to the participants.

The two lotteries in this part pay positive, and almost the same, amounts of money in different states. Thus if a participant is ambiguity averse, s/he may prefer their state-wise mixture of to themselves. In COR, if s/he believes ex ante randomization of RIS hedges ambiguity likewise, she may intentionally choose different alternatives in the two repetitions. However, a participant has no incentive to do so in IND if s/he satisfies Condition 1, which is what Proposition 1 states.

We now test our hypotheses using the data from Part 2. We say a participant is *inconsistent* if s/he chooses different lotteries during the repetitions. If the use of I-RIS improves incentive compatibility, we expect that the proportion of inconsistent participants is smaller in IND than in COR.

Hypothesis 2. *The proportion of participants making inconsistent choices is smaller in IND than in COR.*

Because the two lotteries in each choice situation pay more in different states, behaving inconsistently may hedge ambiguity. Therefore, it is expected that the more randomization seeking (averse) a participant is, the more (less) s/he tends to be inconsistent in COR. Thus, we expect the following:

Hypothesis 3. *In COR, randomization seeking participants are more likely to make an inconsistent choice than randomization averse participants.*

Finally, we consider the treatment effect conditional on randomization attitude. If I-RIS eliminates the opportunitiy to hedge ambiguity, randomization seeking participants would have weaker incentive to behave inconsistently in IND, compared with COR. On the other hand, randomization averse participants would have a weaker incentive to behave consistently in IND than in COR.

Hypothesis 4. *Randomization seeking (resp. averse) participants are more (resp. less) likely to make inconsistent choices in COR than in IND.*

3.2.2 Logistics

We have recruited total of 195 participants from the subject pool of ISER, Osaka University, managed by ORSEE (Greiner, 2015). Among them, 192 (95 in IND, and 97 in COR) completed the two parts of the experiment, which were two weeks a part. We have used Qualtrics (`www.qualtrics.com`) to conduct our online experiment. Registered participants were asked to individually complete the task by clicking the link they received via e-mail within the same day. The link for the experimental site was sent to all the registered participants around 10:00 AM of the date of the experiment. The median of time amounts spent by participants to complete the first and the second part of the experiment are 20 and 3 minutes, respectively.[4] They earned, on average, 979 JPY and 533 JPY in the first and the second part of the experiment including 500 and 300 JPY of participation fees, respectively. Participants received their reward in the form of the amazon gift card, e-mail version.

3.2.3 Results

We now consider a test of Hypothesis 1. For each participant, we tested the null hypothesis that his/her mean of $v_i^{BC} - v_i^{CB} (i = 1, \dots, 10)$ is zero using a nonparametric bootstrap test. As a result, for 52 of the 195 participants, the hypothesis is rejected.[5] Because a nonnegligible number of participants violated the reversal-of-order axiom, Hypothesis 1 is rejected. However, once we pooled all the participants and applied the same test, the hypothesis that the mean is zero is not rejected. This suggests that the aggregate behavior of the participants does not contradict the reversal-of-order axiom, which is consistent with Oechssler et al. (2019).

Result 1. *A nonneglible number of participants violated the reversal-of-order axiom. The aggregate behavior of participants is consistent with the reversal-of-order axiom.*

In IND, 13 out of 95 participants' choices were inconsistent, whereas 17 out of 97 were inconsistent in COR. The fractions of participants making inconsistent choices are not significantly different between the two treatments ($p = 0.5523$, Fisher's exact test). Thus, we reject Hypothesis 2, which states that a smaller fraction of participants making inconsistent choices in IND than in COR.

Result 2. *There is not a significant difference between IND and COR in the proportion of participants behaving inconsistently.*

To test Hypotheses 3 and 4, we focus on the participants with nonneutral randomization attitudes and used logistic regressions to investigate the relationship between the elicited preference indices and inconsistent choices.[6] The dependent variable is a dummy variable that equals 1 if a participant is making inconsistent choices, and 0 otherwise. The set of independent variables includes a dummy variable that equals 1 for COR, and 0 otherwise, a dummy variable that equals 1 if a participant is randomization seeking, and 0 otherwise, as well as interactions between the randomization dummies and COR.

We used the Wald test to understand the relation of randomization attitudes and inconsistency of choices in COR. We found no significant effect of randomization attitudes to inconsistency ($p = 0.4$). Thus Hypothesis 3 is rejected.

Result 3. *In COR, there is no significant difference in the proportion of participants behaving inconsistently between randomization seeking and randomization averse participants.*

We, again, used the Wald test to understand the treatment effect among the subpopulation of randomization seeking participants, and found that the treatment effect is insignificant ($p = 0.46$). The null hypothesis that the coefficient of the COR dummy is zero is not rejected ($p = 0.054$), which shows that the treatment effect for randomization averse participants is also insignificant. Thus, Hypothesis 4 is rejected.[7]

Result 4. *There is no significant difference between COR and IND in the proportion of inconsistent participants among randomization seeking or averse participants.*

[4]In the text, we report the medians of time amounts in the experiment because there are outliers. The means of time amounts are 29 and 7 minutes in the first part and the second part, respectively.

[5]We employed a 5% significance level throughout the paper.

[6]Including randomization neutral participants does not change the following results.

[7]We also estimated a similar model in which the dummy for randomization attitudes is replaced with dummies for ambiguity attitudes. Conducting similar analyses, we obtained results that are consistent with those presented in this and the previous paragraph.

4 Conclusion

We proposed a modification of RIS, named I-RIS, in which participants face ambiguity from different sources in different choice situations. Then, we posed a preference condition that guarantees the incentive compatibility of I-RIS.

We conducted an experiment to test the performance of RIS directly and indirectly and obtained the following results. First, nonneglibile fraction of participants violate the reversal-of-order axiom. Second, we found no significant difference in the performances of RIS and I-RIS. Third, randomization attitudes do not explain inconsistent behaviors under RIS. Fourth, there is no significant difference in performance between RIS and I-RIS among randomization seeking or averse participants. These results suggest that preferences for randomization, which can be motivated by the hedging of ambiguity, do not reduce the reliability of RIS.

References

Anscombe, F. J. and R. J. Aumann (1963) "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, 34 (1), 199–205.

Azrieli, Yaron, Christopher P. Chambers, and Paul J. Healy (2018) "Incentives in Experiments: A Theoretical Analysis," *Journal of Political Economy*, 126 (4), 1472–1503.

——— (2020) "Incentives in Experiments with Objective Lotteries," *Experimental Economics*, 23 (1), 1–29.

Bade, Sophie (2015) "Randomization Devices and the Elicitation of Ambiguity-Averse Preferences," *Journal of Economic Theory*, 159, 221–235.

Becker, Gordon M., Morris H. Degroot, and Jacob Marschak (1964) "Measuring Utility by a Single-Response Sequential Method," *Behavioral Science*, 9 (3), 226–232.

Gilboa, Itzhak and David Schmeidler (1989) "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, 18.

Greiner, Ben (2015) "Subject Pool Recruitment Procedures: Organizing Experiments with ORSEE," *Journal of the Economic Science Association*, 1 (1), 114–125.

Kuzmics, Christoph (2017) "Abraham Wald's Complete Class Theorem and Knightian Uncertainty," *Games and Economic Behavior*, 104, 666–673.

Oechssler, Jörg, Hannes Rau, and Alex Roomets (2019) "Hedging, Ambiguity, and the Reversal of Order Axiom," *Games and Economic Behavior*, 117, 380–387.

Oechssler, Jörg and Alex Roomets (2014) "Unintended Hedging in Ambiguity Experiments," *Economics Letters*, 122 (2), 243–246.

Segal, Uzi (1990) "Two-Stage Lotteries without the Reduction Axiom," *Econometrica*, 58 (2), 349.

Seo, Kyoungwon (2009) "Ambiguity and Second-Order Belief," *Econometrica*, 77 (5), 1575–1605.