An axiomatic theory of dynamic information acquisition *

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Abstract

Consider a decision-maker who has an opportunity to wait for information before making a choice. His behavior generates data of choices and response times that depends on choice situations. How can we explain such data? Aoyama (2019) addresses such a problem and give a characterization result. The present paper summarizes the main result in that paper. The result characterizes a decision-maker who solves an optimal stopping problem, in terms of axioms that impose consistencies between choices and response times. Under the axioms, we elicit the decision-maker's subjective filtration as well as other parameters such as utility, subjective probability, and waiting cost.

Keywords: Response time, Subjective learning, Information aquisition

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1 Introduction

In many economic environments, choice timing is not exogenously fixed, but rather itself is a choice variable of the decision-maker (DM for short). The amount of time consumed to choose, or *response time*, reflects the decision process of DM. This is why behavioral scientists, including recent experimental economists, measure and analyze response time.

The aim of this paper is to axiomatically characterize a model that jointly explains choices and response times. We assume the available behavioral data are choice outcome and response time conditional on the state of the world. From this data, we identify the subjective filtration or a collection of information partitions that gets finer over time. Subjective filtration is not directly observable from the analyst, and it is fixed from the point of DM. Then, imposing axioms, we elicit utility, belief, and cost function, and explain the data as a solution of the optimal stopping problem specified by these parameters as well as the subjective filtration.

Our approach to identifying subjective filtration is simple. If DM would know the realization of some event at a point of time, he uses that information when it is profitable to do so. Thus any information he has must be revealed through behavior when he faces some menu. Therefore, we define his subjective filtration as the smallest one that is necessary to describe his behavior.

The model analyzed here is a dynamic extension of optimal inattention representation (OIR) studied by Ellis (2018). He showed that we can recover the information partitions DM uses from state conditional choice correspondence. Actually, his argument has a slight gap. Though preference for acts has to be non-degenerate so that we can construct a subjective probability, this condition may be violated in his proof. To fulfill this gap, we add a technical axiom *Cyclical Consistency*.

This paper lies in the literature of rational inattention, which studies the implications of costly information acquisition. After Sims (2003) incorporated the idea of rational inattention of economic agent, its axiomatic foundations are studied by Caplin and Dean (2015), de Oliveira et al. (2017), and Ellis (2018). While general models are studied by these papers, there is a rather small number of works that study special cases. This paper takes one step ahead to this direction by considering the case of intertemporal information acquisition.

This paper also relates to the studies of decision-theoretic analysis of response time data. Echenique and Saito (2017) considered a model of DM whose response times reflect the preference intensity. Baldassi et al. (2018) provided an axiomatic foundation of the standard Drift Diffusion Model. Among others, the paper most related to this one is Duraj and Lin (2019). They also characterize a sequential sampling agent using a stochastic choice rule as the primitive. While they assume the information structure of DM is exogenous, here we endogenously elicit the subjective filtration from choice data.

2 Model

2.1 Setup

This subsection introduces the framework. Let Ω be a finite set, which is interpreted as the set of states that describe uncertainty. Let $\mathbb{P}$ be the set of all σ-algebra of Ω. Let X be a convex subset of a metrizable topological vector space and let d be its compatible metric. Let $\mathcal{A}$ be the set of functions from Ω to X. With a natural isomorphism, we regard X as the set of constant function and assume $X \subset \mathcal{A}$. Each element of $\mathcal{A}$, interpreted as a choice, is called an *act*. The set $\mathcal{A}$ is endowed with the uniform metric $d_\infty(f, g) = \sup_{\omega\in\Omega} d(f(\omega), g(\omega))$. Let $\mathcal{K}$ be the set of all non-empty compact sets of $\mathcal{A}$ that is endowed with the Hausdorff metric d_h. Each element of $\mathcal{K}$ is interpreted as a menu of acts. For typical elements of the sets above, we write $\mathcal{F}, \mathcal{G}, \mathcal{H} \in \mathbb{P}$, $x, y, z \in X$, $f, g, h \in \mathcal{A}$, and $A, B, C \in \mathcal{K}$.

Our choice data is a conditional choice correspondence and conditional response time. Conditional choice correspondence is a function $c : \mathcal{K} \times \Omega \to \mathcal{K}$ that satisfies $c(B, \omega) \subset B$ for any $(B, \omega) \in \mathcal{K} \times \Omega$.

Conditional response time is a function $\tau : \mathcal{K} \times \Omega \to \mathbb{R}_+$. For each $B \in \mathcal{K}$, let $P(B)$ be the σ-algebra over Ω generated by $c(B, \cdot)$. For any $B \in \mathcal{K}$, we denote the function $\omega \mapsto c(B, \omega)$ as c_B, and $\omega \mapsto \tau(B, \omega)$ as τ_B.

2.2 Axioms

The axioms we impose to (c, τ) can be classified into three groups. The first group consists of axioms of optimal inattention. These axioms first appeared in Ellis (2018). They are assumptions to guarantee the existence of fundamental preference relation behind the choice correspondence and impose structural assumption on it. The second group consists of axioms of optimal stopping. They are new axioms that describe consistent relations between choice and response time. The third group consists of auxiliary axioms.

2.2.1 Axioms of optimal inattention

The first axiom *INRA* enables us to explain the choice correspondence as a maximization of a consistenct preference relation.

Axiom 1 (*INRA—Independence of Never Relevant Acts*)**.** *If $A \subset B$ and $A \cap c(B, \omega) \neq \emptyset$ for any $\omega \in \Omega$, then $c(A, \omega) = A \cap c(B, \omega)$.*

The next axiom *ACI* is a variant of Independence axiom. This is the empirical content of additive waiting cost.

Axiom 2 (*ACI—Attention Constrained Independence*)**.** *If $\alpha g + (1 - \alpha) f \in c(\alpha g + (1 - \alpha) B, \omega)$, then $\alpha h + (1 - \alpha) f \in c(\alpha h + (1 - \alpha) B, \omega)$*

From c, we can elicit a preference relation $\succeq^R$ over outcomes. For $x, y \in X$, define

$$x \succeq^R y \Leftrightarrow \text{there exists an } \omega \in \Omega \text{ such that } x \in c(\{x, y\}, \omega). \tag{1}$$

Monotinocity axiom states that if an act gives better outcomes at all states than another act, it is more prefered.

Axiom 3 (*M—Monotonicity*)**.** *For $f, g \in B$, if $f(\omega) \succeq^R g(\omega)$ for all $\omega \in \Omega$, then*

$$g \in c(B, \omega) \Rightarrow f \in c(B, \omega). \tag{2}$$

2.2.2 Axioms of optimal stopping

From the data (c, τ), we extract the filtration $\{\mathcal{F}_t\}_{t \in \mathbb{R}_+}$. To do so, first endow $\mathcal{K} \cup \{\emptyset\}$ with the σ-algebra $\sigma(\mathcal{B}(\mathcal{K}) \cup \{\{\emptyset\}\})$, that is, the smallest σ-algebra that contains all the Borel set of $\mathcal{K}$ and the singleton set $\{\emptyset\}$. Next, fixing a menu B, define a function $c_B^t : \Omega \to \mathcal{K} \cup \{\emptyset\}$

$$c_B^t(\omega) = \begin{cases} c(B, \omega) & \text{if } \tau(B, \omega) \leq t \\ \emptyset & \text{otherwise.} \end{cases} \tag{3}$$

We define subjective filtration.

Definition 1. *Subjective filtration is the indexed collection $\mathbb{F} = \{\mathcal{F}_t\}_{t \in \mathbb{R}_+}$ of σ-algebras given by*

$$\mathcal{F}_t = \sigma(c_B^t; B \in \mathcal{K}). \tag{4}$$

That is, $\mathcal{F}_t$ the smallest σ-algebra that makes all of the functions $\{c_B^t\}_{B\in\mathcal{K}}$ measurable. It is easily shown that τ_B is $\mathbb{F}$-adapted for any $B \in \mathcal{K}$. From the subjective filtration and any adapted response time τ, we can define, as usual, the σ-algebra

$$\mathcal{F}_\tau = \{\Delta \in \Sigma | \ \forall t \in \mathbb{R}_+,\ \Delta \cap \{\tau \leq t\} \in \mathcal{F}_t\}, \tag{5}$$

which correspondes to the information DM can obtaine through τ.

The next axiom *DSC* states that choice behavior respects the filtration defined above.

Axiom 4 (*DSC—Dynamic Subjective Consequentialism*)**.** *For $f, g \in B$, $\omega \in \Omega$, and $\Delta \in \mathcal{F}_{\tau_B}$, such that $\omega \in \Delta$, if*

$$f(\omega') = g(\omega') \textit{ for all } \omega' \in \Delta, \tag{6}$$

then

$$f \in c(B,\omega) \Leftrightarrow g \in c(B,\omega). \tag{7}$$

The next axiom *IM* states that if choice from a menu A requires more information than chioce from another menu B, then DM must use more time when he choose from A.

Axiom 5 (*IM—Information Monotonicity*)**.** *If $P(A) \subset P(B)$, then $\tau_A \leq \tau_B$.*

The next axiom *TI* states that response time depends only on relative values of outcomes brought by acts in the menu.

Axiom 6 (*TI—Time Invariance*)**.**

$$\tau_{\alpha f+(1-\alpha)B} = \tau_{\alpha g+(1-\alpha)B} \tag{8}$$

2.2.3 Auxiliary axioms

Define a relation

$$A \succeq^D B \Leftrightarrow \text{For all } \omega \in \Omega,\ A \cap c(B,\omega) \neq \emptyset. \tag{9}$$

The relation $A \succeq^D B$ means that, facing A, DM can emurate the optimal policy he uses if B is given. And thus A is directly selected over B. Let $\succeq^I$ denotes the transitive closure of $\succeq^D$.

The next axiom *CC* requires a consistency of the relation on menus $\succeq^I$ and the relation on outcomes $\succeq^R$.

Axiom 7 (*CC—Cyclical Consistency*)**.** *For $x, y \in X$, if $\{x\} \sim^I \{y\}$, then $x \sim^R y$.*

The next axiom requires unboundedness of expected utility function. This is necessary to calibrate the cost function.

Axiom 8 (*U—Unboundedness*)**.** *There exist $x, y \in X$ such that $x \succ^R y$ and, for any $\beta \in (0,1)$, there exist $z^*, z_* \in X$ such that*

$$\beta z^* + (1-\beta)y \succ^R x,\ \ y \succ^R \beta z_* + (1-\beta)x. \tag{10}$$

The next axiom requires a form of upper hemi-continuity of c.

Axiom 9 (*C—Continuity*)**.** *For any $\omega \in \Omega$, $\{B_n\}_{n=1}^\infty$ and $\{f_n\}_{n=1}^\infty$ such that $B_n \to B$ and $f_n \to f$ with $f_n \in c(B_n,\omega)$ and $f \in B$, if*

$$P(B_n)(\omega) = \mathcal{F}_{\tau_B}(\omega) \textit{ for any } n \in \mathbb{N}, \tag{11}$$

then

$$f \in c(B,\omega). \tag{12}$$

2.3 Representation result

Write the set of $\{\mathcal{F}_t\}_{t\in\mathbb{R}_+}$-adapted stopping time as $\mathcal{T}$. Now we state the sufficiency for the axiom for the date to be explained by the optimal stopping representation.[1]

Theorem 1. *If c and τ satisfy INRA, ACI, M, DSC, IM, TI, CC, U, and C, then there exists (u, π, γ) such that*

$$\tau_B \in \arg\max_{\tau\in\mathcal{T}} E[\max_{f\in B} E[u(f)|\mathcal{F}_\tau]] - \gamma(\tau), \tag{13}$$

$$c(B, \omega) = \arg\sup_{f\in B} \mathrm{E}[u(f)|\mathcal{F}_{\tau_B}](\omega)\ \pi\text{-}a.s. \tag{14}$$

2.4 Proof sketch

Here we provide an outline of the proof of the representation result above. First, we note that *ACI* implies Independence axiom over X and obtain an expected utility function defined on X. Thus we transform each act into a utility act.

Then, we turn to the incomplete relation $\succeq^I$ over menus. *INRA* implies the transitivity of $\succeq^I$. *M* implies its monotonicity; if any acts in a menu are dominated by some acts in another menu state by state, the latter is preferred. Finally, *ACI* implies translation invariance of $\succeq^I$; if $A \succeq^I B$, then $A + f \succeq^I B + f$. In order to elicit a subjective filtration, we focus on singleton menus f and consider a set $K = \{f \mid f \succeq^I 0\}$. *CC* and *U* implies that we can separete K from 0. We apply the separating hyper-plane theorem to obtain a subjective probability.

Next, we consider functions $F : \Omega \to \mathcal{A}$, called *plan*. When DM confronts a menu, he considers how much time he will wait and what to choose contingent on the obtained information. In other words, he chooses a pair (F, τ) of a plan and response time. Thus we consider a preference relation over such pairs. For each pair (F, τ), we construct a menu B^F_τ such that, facing B^F_τ, DM would wait according to τ and choose according to F. That is, when he faces B^F_τ and ω realizes, he will wait until $\tau(\omega)$ and choose $F(\omega) \in B$. From $\succeq^I$, we can define such preference;

$$(F, \tau) \succeq (G, \sigma) \Leftrightarrow B^F_\tau \succeq^I B^G_\sigma. \tag{15}$$

IM implies that DM prefers to wait less time, and *TI* implies additive cost for response time. Using these properties, we obtain the representation through a variational argument.

3 Conclusion

This paper studied a model of DM has an opportunity to wait for information before making a choice. We facilitated a new primitive to identify a subjective filtration and represented the data as a solution of an optimal stopping problem. Note that most of the existing studies exploited recursive structure of choice alternative to elicit subjective filtration. Here we used response time data to complement choice data. This enabled us to work on a non-recursive environment.

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[1]A similar result also holds when we assume the state-space is an arbitrary measurable space. In that case, the representation of response time is exactly the same as the finite state case. However, the representation of chioce correspondence becomes a slightly weaker form.

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