Attribute-Based Inferences and Random Limited Consideration: A Representation Result

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Abstract

This paper presents an axiomatic model of random limited considerations under attribute-based inferences. To characterize the model, this paper studies a preference for commitment stemming from procedural costs on choosing from menus. The key axiom characterizes a model of random limited consideration, which is an extended and generalized version of the reference-dependent choice model of Ok et al. (2015) in the sense that (i) consideration sets are formed under attribute-based inferences randomly, and that (ii) reference points themselves can be chosen from menus. The two extensions make it possible to allow for not only the *Attraction Effects*, but also the *Compromise Effects*.

Keywords: Limited Consideration, the Attraction Effect, the Compromise Effect, Salience, Mistakes/Errors.

JEL classification: D01, D90, D91.

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1. Introduction

This paper develops a model of random limited considerations under attribute-based inferences. The present model is an extended and generalized model of limited attention in Ok et al. (2015) in the sense that consideration sets are formed with attribute-based inferences randomly, and that reference points themselves can be chosen options. The two extensions make it possible to capture not only the *Attraction Effects*, but also the *Compromise Effects*.[2]

1.1. Motivation

The marketing literature such as Hoyer (1984), Hauser and Wernerfelt (1990), Roberts and Lattin (1991), etc. supports that consumers deliberately form consideration sets, and then choose an alternative from consideration sets. Recently, in decision theory, theories of revealed attention have been developing (Masatlioglu et al., 2012). In addition, the study of random random consideration sets, which leads to random choices, has been developing (see the subsection of related literature). There are various reasons why decision makers randomly have consideration sets such as cognitive abilities, procedural costs, naivete, etc.

This paper studies choices from random consideration sets by studying the effect of procedural costs in decision-making processes. The decision maker has a single utility function that evaluates options, but before choosing an option from a menu, the decision maker (consciously or unconsciously) narrows down some options from the menu randomly by using attribute-based inferences.

The contributions of this paper are two-fold. First, by taking preferences over menus as primitives, we introduce plausible axioms on deliberate limited considerations, and provide an axiomatic foundation for it. In the framework of preferences over menus, the decision maker prefers larger menus (*preferences for flexibility*) or smaller menus (*preferences for commitment*) under plausible situations.[3] We study a type of *commitment preferences*, since the reason why decision makers do not consider all feasible alternatives stems from limited cognitive abilities such as procedural costs, psychological effects, etc.

Second, by extending the reference-dependent choice model in Ok et al. (2015), we explain about both the *Attraction Effect* and the *Compromise Effect*. Following Ok et al. (2015), in this model, an alternative in a menu is regarded as a reference point, but analysts do not observe which alternative is a reference point. Then, the resulting behaviors seem to be random. Moreover, this irrelevant alternative as a reference point affects resulting behaviors, which leads to violations of WARP. Especially, we allow options as reference points to be chosen. This generalization in the model can lead to both the *Attraction Effect* and the *Compromise Effect*.

1.2. A Preview of Results

Let us denote notation. Let X be a finite set. Let $\Delta(X)$ be the set of all probability distributions over X, endowed with the weak convergence topology. The elements in $\Delta(X)$ are interpreted as *options*, denoted by $p, q, r \in \Delta(X)$. Let $\mathscr{A}$ be the set of all non-empty compact subsets of $\Delta(X)$, endowed with the Hausdorff topology. The elements in $\mathscr{A}$ are called *menus*, denoted by $A, B, C \in \mathscr{A}$.

We investigate a decision maker whose choice consists of two stages. In the first stage, the decision maker chooses a set of options, i.e., a menu. In the second stage, the decision maker chooses an option from the set that she chose at the first stage.

At the second-stage choices, choosing one option from menus is a difficult task. The decision maker could anticipate that she will feel cognitively demanding at that stage, as menus

[2] Ok et al. (2015) allows for only the *Compromise Effect*.

[3] In the presence of *unforeseen contingencies*, the decision maker exhibits *preferences for flexibility* (Kreps (1992)). On the other hand, in the presence of *temptation*, *shame*, or *regret*, the decision maker exhibits *preferences for commitment* (Gul and Pesendorfer (2001), Dillenberger and Sadowski (2012), Sarver (2008)).

are getting larger. In light of such a potential ex-post feeling, the decision maker chooses a menu at the first stage.

When the decision maker chooses a menu at the first stage, she maximizes the following utility function. There exists a four-tuple $\langle u, (\mathscr{U}_A)_{A\in\mathscr{A}}, \mu, c\rangle$ where $u : \Delta(X) \to \mathbb{R}$ is a utility function, $(\mathscr{U}_A)_{A\in\mathscr{A}}$ is a profile of the sets of the non-empty set of real-valued attribute functions $U : X \to \mathbb{R}$, μ is a probability distribution over $\Delta(X \cup \{\Diamond\})$,[4] and $c : 2^{\Delta(X)}\backslash\{\emptyset\} \to \mathbb{R}$ is a monotone function with $c(S) \geq 0$ for all $S \in 2^{\Delta(X)}\backslash\{\emptyset\}$ and $c(\{p\}) = 0$ for all $p \in \Delta(X)$.

The utility of a menu A is as follows:

$$V(A) = \sum_{q\in \mathrm{supp}(\mu_A)} \mu_A(q)\Big[\max_{p\in\mathscr{U}_A^{\uparrow}(q)} u(p) - c\big(\mathscr{U}_A^{\uparrow}(q)\big)\Big], \tag{1}$$

where $\mathscr{U}_A^{\uparrow}$ is the consideration set of the menu A, defined by

$$\mathscr{U}_A^{\uparrow}(q) := \{p \in A \mid \mathbb{E}_p(U) \geq \mathbb{E}_q(U) \text{ for every } U \in \mathscr{U}_A\}, \tag{2}$$

and

$$\mathrm{supp}(\mu_A) := \{q \in A \mid \rho(q) > 0,\ q \in A \cup \{\Diamond\}\}. \tag{3}$$

We explain about the model. First, since considering all alternatives is cognitively demanding, the decision maker forms a consideration set $\mathscr{U}_A^{\uparrow}$, given a (chosen) menu A. The decision maker chooses a *best* one from the consideration set with its procedural cost. Such consideration sets formation takes procedural costs. This is described by the cost function c. Since the cost function is monotone, i.e., for all $S, T \in 2^{\Delta(X)}\backslash\{\emptyset\}$ with $T \subset S$, $c(S) \geq c(T)$, considering a lot of options needs more thinking. Next, in this model, a consideration set is formed based on an alternative in the menu; that is, $\mathscr{U}_A^{\uparrow}$ depends on it. Moreover, this model allows consideration sets to be random; that is, some alternatives in the menu can be reference points, and then the consideration set is formed. The probability that an alternative $q \in A$ is a reference point is denoted by $\mu_A(q)$. Thus, this utility function produces random choices.

1.3. Related Literature

We provide a brief literature review. For a limitation, we do not provide the review of the literature on preferences over menus. This paper is an extended model of Ok et al. (2015) in which the consideration set induced by an option as a reference point is random, and its reference point can be chosen. To do so, some conditions in Ok et al. (2015) are relaxed.

In revealed preference theory, Masatlioglu et al. (2012) is the first to characterize limited attention in the case of deterministic choices. More generally, Cattaneo et al. (2018) also develop a random attention model (RAM), by using the revealed preference approach. Gul et al. (2014) presents a random attribute rule. Manzini and Mariotti (2014) develop a random consideration set rule. In a different viewpoint, Caplin et al. (2018) studies optimal consideration set formations in terms of rational inattention.

2. Axioms and Result

2.1. Axioms

The primitive of this paper is a binary relation $\succeq$ over $\mathscr{A}$. The asymmetric and symmetric parts are denoted by $\succ$ and $\sim$, respectively.

Axiom 1 (Weak Order): $\succeq$ is *complete* and *transitive*.

Axiom 2 (Mixture Continuity): For any $A, B, C \in \mathscr{A}$, $\{\lambda \in [0,1] \,|\, \lambda A + (1-\lambda)B \succeq C\}$ and $\{\lambda \in [0,1] \,|\, C \succeq \lambda A + (1-\lambda)B\}$ are closed.

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[4] The symbol $\diamond$ is a generic object that does not belong to $\Delta(X)$.

Definition 1. For any $A \in \mathscr{A}$ with $|A| > 1$ and $p \in A$, we say that, for any $q \in A$, $p \succsim_A q$ if
$$A \succeq A \backslash \{p\}.$$

This definition says that if a menu A is weakly preferred to the menu that an alternative p is removed from the menu A, i.e., $A \succeq A \backslash \{p\}$, then p is weakly preferred to q for all q in A in terms of menu-dependent preferences over alternatives.

Axiom 3 (Menu-Dependent Preferences): For any $A \in \mathscr{A}$ with $|A| > 1$, $\succsim_A$ is (i) *reflexive*, (ii) *transitive*, (iii) *continuous*, and (iv) *affine*.

Axiom 4 (Menu-Dependent Commitment): For any $A \in \mathscr{A}$ with $p \in A$, if $p \succsim_A q$ for any $q \in A$, then $\{p\} \succeq A$.

This axiom is included into a class of *commitment preferences*, and the axiom says that if p is weakly preferred to q for all q in A in terms of menu-dependent preferences over alternatives, the singleton menu of p is weakly preferred to the menu A. The intuition behind the axiom is that even though p is the $\succsim_A$-best in the menu A, choosing p from A is cognitively demanding due to some procedural aspects such as reading the alternatives on the menu.

Axiom 5 (Thinking Aversion): For any $A \in \mathscr{A}$, $p, r \in \Delta(X)$, and $\lambda \in [0,1]$,
$$\{p\} \succeq A \Rightarrow \lambda\{p\} + (1-\lambda)\{r\} \succeq \lambda A + (1-\lambda)\{r\}.$$

This axiom is a weaker version of *Singleton Independence*: for any $A \in \mathscr{A}, p, r \in \Delta(X)$, and $\lambda \in [0,1], A \succeq B \Rightarrow \lambda A + (1-\lambda)\{r\} \succeq \lambda B + (1-\lambda)\{r\}$. The interpretation of this axiom is as follows. Suppose $\{p\} \succeq A$. We do not suppose that the decision maker has no temptation-driven preference. It is inferred that this ranking stems from the procedural cost on choosing an alternative from the menu A. The singleton $\{p\}$ has no thinking cost. Then, the axiom of *Independence* holds under the mixture of singletons.

2.2. Result

We state the main result.

Theorem 1. *The following statements are equivalent.*
(a) $\succeq$ *satisfies Axioms 1 - 5.*
(b) There exists a four-tuple $\langle u, (\mathscr{U}_A)_{A\in\mathscr{A}}, \mu, c\rangle$ *where*
- *(i)* $u : \Delta(X) \to \mathbb{R}$ *is a non-constant linear function;*
- *(ii)* $(\mathscr{U}_A)_{A\in\mathscr{A}}$ *is a profile of the sets of the non-empty set of real-valued functions* $U : X \to \mathbb{R}$,
- *(iii)* μ *is a probability distribution over* $\Delta(X \cup \{\diamondsuit\})$*, and*
- *(iv)* $c : 2^{\Delta(X)}\backslash\{\emptyset\} \to \mathbb{R}$ *is a monotone function with* $c(S) \geq 0$ *for all* $S \in 2^{\Delta(X)}\backslash\{\emptyset\}$ *and* $c(\{p\}) = 0$ *for all* $p \in \Delta(X)$

such that $\succeq$ *is represented by* $V : \mathscr{A} \to \mathbb{R}$ *defined by*
$$V(A) = \sum_{q \in \mathrm{supp}(\mu_A)} \mu_A(q)\Big[\max_{p \in \mathscr{U}_A^{\uparrow}(q)} u(p) - c\big(\mathscr{U}_A^{\uparrow}(q)\big)\Big],$$
where $\mathscr{U}_A^{\uparrow}$ *is the consideration set of the menu A, defined by*
$$\mathscr{U}_A^{\uparrow}(q) := \{p \in A \mid \mathbb{E}_p(U) \geq \mathbb{E}_q(U) \text{ for every } U \in \mathscr{U}_A\},$$
and
$$\mathrm{supp}(\mu_A) := \{q \in A \mid \rho(q) > 0,\ q \in A \cup \{\diamondsuit\}\}.$$

2.3. Proof Outline of Theorem 1

We provide the proof overview of the sufficiency part in Theorem 1. Let $\mathscr{A}_s \subset \mathscr{A}$ be the set of *singletons*. First, by mainly using the axioms of *Weak Order* and *Mixture Continuity*, we can apply the result in the vNM-type expected utility theorem; that is, for each $p \in \Delta(X)$, $V(\{p\}) = u(p)$ for some $u : \Delta(X) \to \mathbb{R}$, which represents $\succeq$ on $\mathscr{A}_s$. Next, we consider the case of *non-singletons*. By Definition 1, we have $(\succsim_A)_{A \in \mathscr{A}}$ with $|A| > 1$. By the axiom of , for each menu $A \in \mathscr{A}$ with $|A| > 1$, we obtain a set of real-valued functions on X denoted by $\mathscr{U}_A$ (Dubra et al., 2004). Let us denote the following: For each $p \in A$, $\mathbb{E}_p(U)$ for each $U \in \mathscr{U}_A$. We consider a binary relation $\succeq^*$ on $\mathscr{A}$ that can be represented by a (*random*) *Strotz-type* representation. By using the axiom of *Menu-Dependent Preferences* and *Menu-Dependent Commitment*, $\succeq^*$ is represented by

$$V_q^*(A) = \max_{p \in \mathscr{U}_A^{\uparrow}(q)} u(p), \tag{4}$$

for some $q \in A$. By the axioms in Theorem 1, for any $A \in \mathscr{A}$, there exits $p_A \in \Delta(X)$ such that $\{p_A\} \sim A$. Let $V'(A) = V'(\{p_A\}) = v(p_A)$. Define $c : 2^{\Delta(X)} \backslash \{\emptyset\} \to \mathbb{R}$ by

$$c\big(\mathscr{U}_{\mathscr{A}}^{\uparrow}(q)\big) := V_q^*(A) - V'(A). \tag{5}$$

We can show that c is monotonic by using the axiom of *Menu-Dependent Commitment* and *Thinking Aversion*. Finally, by using the axiom of *Thinking Aversion* and applying the *mixture state space theorem*, we obtain the following. For any $A \in \mathscr{A}$,

$$V(A) = \sum_{q \in \mathrm{supp}(\mu_A)} \mu_A(q) \Big[\max_{p \in \mathscr{U}_A^{\uparrow}(q)} u(p) - c\big(\mathscr{U}_A^{\uparrow}(q)\big) \Big],$$

where $\mu_A \in \Delta(A \cup \{\diamondsuit\})$ and $\mathrm{supp}(\mu_A) := \{q \in A \mid \rho(q) > 0,\ q \in A \cup \{\diamondsuit\}\}$.

3. Discussions and Concluding Comments

3.1. Ex-Post Choices

We study the ex-post random choices of the utility representation. To do so, let us denote the tie-breaking rule by $\tau : \Delta(X) \to [0,1]$, and given a consideration set $\mathscr{U}_A^{\uparrow}(q)$ for some $q \in A$, $\sum_{p \in \arg\max \mathscr{U}_A^{\uparrow}(q)} \tau\Big(p \mid \arg\max \mathscr{U}_A^{\uparrow}(q)\Big) = 1$. The ex-post random choice rule says that the probability of choosing an alternative from the menu is described in the following way:

$$\rho(p \,|\, A) = \sum_{q \in \mathrm{supp}(\mu_A)} \mu(q) \cdot \tau\Big(p \mid \arg \max_{p \in \mathscr{U}_A^{\uparrow}(q)} u(p)\Big). \tag{6}$$

If the tie breaking rule gives each choosable alternative the equal probability, then the random choice rule is described in the following way:

$$\rho(p \,|\, A) = \sum_{q \in \mathrm{supp}(\mu_A)} \mu(q) \cdot \frac{1}{|\{p \in A \,|\, p \in \arg\max_{p \in \mathscr{U}_A^{\uparrow}(q)} u(p)\}|}. \tag{7}$$

We explain about the interpretation of the ex-post choices. The choice probabilities of an alternative are determined by how frequently the alternative is best in the consideration set.

3.2. Concluding Comments

In this paper, we have axiomatized a utility representation in which a consideration set is randomly formed under attribute-based inferences, by extending the representation result in Ok et al. (2015). We generalize their model, and the novelties of this paper are as follows. First, in the same way as Ok et al. (2015), some options in menus can be reference points. In general, however, all options can be reference points stochastically. their study is limited in the case of deterministic cases, but we allow options to be reference points randomly. Second, we allow chosen options themselves to be reference points. Unlike Ok et al. (2015), this extension can allow for the *Compromise Effect*.

Axiomatically, in the framework of preferences over menus, we have introduced a type of *commitment preferences*. Our axioms allow a type of (random) Strotz-type utility representation, and it is shown that a type of the reference-dependent choice model of Ok et al. (2015) can be described as a (random) *Strotz-type* utility representation. In the similar way as Ok et al. (2015), we elicit a (menu-dependent) attribute space by applying the result in Dubra et al. (2004). One remark is that, to provide an axiomatic foundation, we have used a richer framework (lottery domains) than that of Ok et al. (2015).

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