Regret-CAPM: A Model of Regret and Asset Pricing

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Abstract

This paper examines the effect of regret on asset pricing in a model where each individual compares the return on his chosen portfolio with a countfactual, the return on an unchosen portfolio. We derived a single beta asset pricing formula where an asset’s expected rate of return is increasing to its beta with respect to the difference between the market average return and the market-wide average countfactual. In equilibrium, a positive excess return is paid as a premium for regret aversion. CAPM undervalues (overvalues) assets with positive (negative) correlations to the average countfactual.

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1. Introduction

Regret is a painful feeling caused by “counterfactual thinking” that compares the true outcome of a choice with a counterfactual outcome, that is, with “what might have been”. Regret has strong effect on financial decision making. For example, Nobel Prize winner Harry Markowitz described how anticipated regret affected his choice of a pension plan. “I should have computed the historical co-variance of the asset classes and drawn an efficient frontier. Instead, I visualized my grief if the stock market went way up and I wasn’t in it – or if it went way down and I was completely in it. My intention was to minimize my future regret. So I split my contributions 50/50 between bonds and equities. (As quoted in Zweig (2007), pp 4.)”

Since the portfolio selection theory developed by Markowitz (1952) and the capital asset pricing model (CAPM) by Sharpe (1964), the mainstream asset pricing theories generally assume that rational investors’ decision making is not affected by emotions. It is only in recent years that researchers started to examine the effect of regret on investor behavior in financial markets. For example, Braun and Muermann (2004) show that regret can affect individuals’ insurance purchase decisions. Fogel and Berry (2006) examine the relation between regret and the disposition effect. Muermann et al (2006) analyze regret adverse individuals’ asset allocation decisions in a defined contribution pension plan. Michenaud and Solnik (2008) examine the effect of regret aversion on currency hedging decisions. Qin (2015) shows that regret aversion can cause bubbles and crashes in financial markets. Strahilevitz et al (2011) and Frydman and Camerer (2016) show that regret can affect the repurchase of stocks previously sold. Nevertheless, the number of studies is still small and, to the author’s knowledge, asset pricing model under regret aversion has not been developed yet.

As a first step to introduce regret aversion into asset pricing, the present paper constructs a model with regret adverse individuals. Following the Regret Theory developed by Bell (1982, 1983) and Loomes and Sugden (1982, 1987), we assume that each individual has a “modified utility function” that includes both the utility of realized return and the disutility of regret. Except for the assumption about regret aversion, the setting of the model is similar to CAPM and many other asset pricing models: in the market multiple risky assets and a riskless asset are traded among multiple individuals, where the returns of risky assets are multivariate normally distributed. In this setting, if individuals have concave utility functions, then, as the standard asset pricing theory has suggested, two-fund separation will hold and, consequently, the single market beta pricing formula of CAPM can be derived in equilibrium. However, when individuals are regret adverse as assumed by Regret Theory, two-fund separation no more holds. This fact raises a simple but important question about asset pricing: when there are regret adverse individuals in the market, how equilibrium prices will be?

The present paper explores this question in a model where each individual compares the return on his chosen portfolio with a countfactual that is the return on an unchosen portfolio. If the chosen portfolio underperforms the countfactual, the individual feels regret; otherwise, he feels rejoice. The effect of regret or rejoice on utility is measured by an increasing and concave function, which is called “regret function” following previous studies. Both the countfactual and the functional of the regret function can differ among individuals. In equilibrium, the effects of individuals’ regret aversion are aggregated. A single beta pricing formula holds where a risky

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2 For a discussion about two fund separation, see Huang and Litzenberger (1998).
asset’s expected return is increasing to its “regret beta”, which is a beta defined with respect to the gap between the market average return and the market-wide average countfactual.

We first derive this price formula in the case where individuals are regret averse but are risk neutral. In this case, the modified utility function takes the same from as the original modified utility function of Regret Theory, which was first proposed by Bell (1982) and Loomes and Sugden (1982).\(^3\) We show that a single beta pricing formula holds in equilibrium. This formula is very similar to that of CAPM, but the beta is not a market beta, but a “regret beta”. We then analyze the case where individual are both risk averse and regret averse. In this case, a single regret beta pricing formula still holds, but the average countfactual is adjusted by terms that reveal the magnitude of risk aversion.

The asset pricing formula proposed in the present study not only gives a theoretical explanation to the effect of regret on equilibrium asset prices, but also provides some important implications. First, on average, the market provides a positive excess return as a premium to regret aversion. Second, if CAPM is applied to a market where some individuals are regret averse, assets will be systemically mispriced: assets with positive correlations to the average countfactual, calculated after adjusted for market risk, will be undervalued, while those with negative correlations will be overvalued.

The present paper provides the first asset pricing model under regret aversion. Although both the setting and the methods of the model are very simple, it made a first step to introduce regret into asset pricing theory, the foundation of modern finance theory. In this sense, this paper contributes to the literature on asset pricing as well as to the literature of Regret Theory.

The rest of the paper is organized as follows. Section 2 describes the setting of model and analyzes the market equilibrium where individuals are regret averse but risk neutral. Section 3 observes the case where individuals are both regret averse and risk averse. Section 4 is conclusion.

2. Asset pricing under regret aversion

In the market \( I \) individuals \((i = 1, 2, ..., I)\) trade \( N \) risky assets \((j = 1, 2, ..., N)\) and one riskless asset \((j = 0)\), where the number of outstanding shares of each risky asset is normalized to one, while the net supply of the riskless asset is zero. For each risky asset \( j \), let \( P_{j,0} \) denote the price at date 0, \( P_{j,1} \) the price of at date 1, and \( r_j \equiv \frac{P_{j,1}}{P_{j,0}} - 1 \) the rate of return. \( \{P_{j,1}\} \) are multivariate normally distributed. \( r_f \) denotes the risk free rate which is a constant. At \( t = 0 \), individual-\( i \) allocates his initial wealth \( W_{i,0} \) among assets. At \( t = 1 \), all positions are liquidated. With \( w_{i,j} \) denoting the proportion of wealth that individual-\( i \) invests on asset-\( j \), the rate of return on his portfolio is

\[
l_i \equiv r_f + \sum_{j=1}^{N} w_{i,j}(r_j - r_f)
\]

and his terminal wealth is \( W_{i,1} = W_{i,0}(1 + l_i) \).

Following Bell (1982) and Loomes and Sugden (1982), we assume that an individual will compare the return on his portfolio to a counterfactual, which will result in regret or rejoice. More specifically, we assume that individual-\( i \) has a “modified utility function”

\(^3\) The original Regret Theory aims to explain the Allais Paradox and other paradoxes in rational decision making theories. Until 1990s, Regret Theory had only limited influence in economics, especially compared with the Prospect Theory. However, this theory caught a lot of attentions in neuroscience and psychology, where researchers are interested in the relation between emotions on decision making. A vast number of experimental studies, such as the influential studies of Camille et al (2004) and Coricelli et al (2005), provide supportive evidence for the Regret Theory. Recently, Steiner and Redish (2014) conducted an experiment that shows regret affects rat’s behavior as well. For a review, see Coricelli et al (2007) and Crespi et al (2012).
\[ V_i(W_{i,1}) = W_{i,1} + f_i(W_{i,1} - H_i), \]  \hspace{1cm} (2)

where \( f_i' > 0, \ f_i'' < 0, \ f_i(0) = 0, \) and \( E\left[f_i''(W_{i,1} - H_i)\right] < \infty. \)

\( H_i \) represents the counterfactual that is used by individual-\( i \) to compare with the return on his chosen portfolio. The assumption of the modified utility function in equation (2) is an extension to the original modified utility function proposed by Bell (1982) and Loomes and Sugden (1982). These researchers examine the effect of regret on an individual’s choice between two assets; thus, the modified utility function proposed by them is pairwise:

\[ u(r_j, r_k) = r_j + f(r_j - r_k), \]  \hspace{1cm} (3)

\[ u(r_k, r_j) = r_k + f(r_k - r_j) \]  \hspace{1cm} (4)

where \( r_j \) is the uncertain outcome of asset-\( j \), \( r_k \) is the outcome of asset-\( k \), and \( f(\cdot) \) is a strictly increasing concave function with \( f(0) = 0 \). In their model, if an individual chooses asset-\( j \), the countfactual is the return on the unchosen asset; as a result, his utility is increasing in \( r_j \), the return on the chosen asset, and is decreasing in \( r_k \), the counterfactual return. The function \( f(\cdot) \) measures the individual’s regret or rejoice and is named as “regret-rejoice function” by Loomes and Sugden (1982).

Different to the original model of Regret Theory, the present paper considers a setting where there are multiple assets. Furthermore, an individual needs not choose one particular asset; instead, he can diversify his wealth among assets. Therefore, the assumption about countfactual needs to be extended. We assume that an individual’s contractual is

\[ H_i = W_{i,0} \]  \hspace{1cm} (2)

where \( h_i \) is the rate of return on a portfolio. For example, an individual may compare the actual investment return with the forgone interest he would receive if he put all his money in a saving account. In such a case, \( h_i = r_f \) and \( H_i = W_{i,0} (1 + r_f) \). If \( W_{i,1} < H_i \), the individual feels regret; if \( W_{i,1} > H_i \), he feels rejoice. Another example of countfactual may be the market portfolio return, which can be easily obtained by investing in stock market index ETFs. In this case, \( h_i = r_m \), where \( r_m \) denotes the rate of return on the market portfolio.

Besides the riskless asset and the market portfolio, an individual can also choose other asset or portfolio as a countfactual. Thus, in our model, the number of selectable countfactual is numerous and we do not require individuals to choose a particular countfactual. This is a generalization to the assumptions of previous studies.

Using the notations of true return \( l_i \) and countfactual return \( h_i \), the modified utility function in equation (2) can be expressed in the following way:

\[ V_i(W_{i,1}) = W_{i,1} + f_i \left(W_{i,1}(l_i - h_i)\right). \]  \hspace{1cm} (5)

Each individual trades to maximize the expectation of his modified utility \( E[V_i(W_{i,1})] \). By the Stein’s Lemma, the first order condition for this problem is

\[ E[r_j] - r_f = \frac{1 + E[f_i'(W_{i,0}(l_i - h_0))]}{E[f_i''(W_{i,0}(l_i - h_0))]} \cdot cov(l_i - h_i, r_j). \]  \hspace{1cm} (6)

The market clearing condition is

\[ \sum_{i=1}^{N} w_{ij} W_{i,0} / P_{j,0} = 1, \ j = 1,2,...,N; \]  \hspace{1cm} (7)

\[ \sum_{i=1}^{N} w_{i0} W_{i,0} = 0. \]  \hspace{1cm} (8)

An equilibrium is defined as a profile of trading strategy \( \{w_{ij}\} \) that satisfies the first order condition as well as the market clearing condition. By aggregating the first condition across individuals, the following equation is obtained.

\[ E[r_j] - r_f = \frac{-M_0}{\sum_{i=1}^{N} w_{i0} W_{i,0} / P_{j,0}} \cdot cov(r_m - h_m, r_j) \]  \hspace{1cm} (9)

where \( r_m \equiv \frac{M_1}{M_0} - 1 \) and \( h_m \equiv \sum_{i=1}^{N} \frac{w_{i0}}{M_0} h_i. \)

Here, \( r_m \) is the rate of return of the market portfolio. Note that the riskless asset has zero net supply. When the market clearing condition in equations (7)-(8) holds, the average rate of
return among individuals equals to the rate of return on the market portfolio of risky assets. That is, \( r_m = \sum_{i=1}^{I} \frac{\mathbf{W}_i}{\mathbf{M}_0} t_i \). By definition, \( h_m \) is the marked-wide average countfactual. The term \( r_m - h_m \) is the gap between actual return and countfactual return at aggregate level. Roughly speaking, if \( r_m - h_m < 0 \), individuals feel regret on average; if \( r_m - h_m > 0 \), they feel rejoice on average. Equation (9) shows that an asset’s excess return \( E[r_j] - r_f \) is determined by covariance \( \text{cov}(r_m - h_m, r_j) \). Recall that in CAPM, an asset’s excess return is determined by a market factor, which result leads to the single market beta asset pricing formula. Analogous to CAPM, in our model a single beta formula can be derived too, but the beta is not a market beta, but a “regret beta.” The proposition below formally addresses this result.

Proposition 1. If \( \text{var}(r_m - h_m) > 0 \), then,
\[
E[r_j] - r_f = \beta_{jg} (E[r_m] - E[h_m])
\]
where \( \beta_{jg} \equiv \frac{\text{cov}(r_j, r_m - h_m)}{\text{var}(r_m - h_m)} \) and \( j = 1, 2, \ldots, N \). Moreover,
\[
E[r_m] - E[h_m] > 0.
\]

Proposition 1 provides a signal beta asset pricing formula. By equation (14)-(11), a risky asset’s excess return \( E[r_j] - r_f \) is increasing to \( \beta_{jg} \). Here, \( \beta_{jg} \) can be called a “regret beta” in the sense that it is the beta with respect to the gap between the market average return and market-wide average countfactual. The result in equation (11) comes from the concavity of the regret function. The intuition is clear: in order to ensure that investors wish to hold the outstanding shares of the risk assets, on average they should be able to obtain rejoice, not regret. In this sense, the term \( E[r_m] - E[h_m] \) is the “regret premium” paid by the asset market.

Next, we observe the case of \( h_m = r_m \), where the average counterfactual equals to the market return. By equation (9), the following result holds.

Proposition 2. If \( h_m = r_m \), then, \( E[r_j] = r_f \) for all \( j \).

Note that the average return that individuals receive from the asset market is the market portfolio return \( r_m \), while the market wide average countfactual is \( h_m \). If \( h_m = r_m \), there is neither regret nor rejoice at aggregate level. Consequently, the market can be in equilibrium only when \( E[r_j] = r_f \) for all \( j \). A special case of \( h_m = r_m \) is that \( h_i = r_m \) for all \( i \), which means that all individuals compare their realized returns with the market portfolio.

Another case worth particular attention is \( h_m = r_f \), where on average individuals’ counterfactual equals to the riskfree rate. By Proposition 1, the following result holds.

Proposition 3. If \( h_m = r_f \), then,
\[
E[r_j] - r_f = \frac{\text{cov}(r_j, r_m)}{\text{var}(r_m)} (E[r_m] - r_f)
\]

This pricing formula is the same as the one of CAPM. However, it is derived under different assumptions. In CAPM, individuals are risk averse and feel no regret; in Proposition 3, individuals are risk neutral but feel regret/rejoice. The intuitions are also different. The intuition of CAPM is that only un-diversified risk matters in equilibrium. In contrast, the intuition of Proposition 3 is that that when \( h_m = r_f \), individuals’ regret/rejoice at aggregate level depends on market return \( r_m \); as a result, a risky asset’s relation to the market return becomes the single factor that determines its price. A special case of \( h_m = r_f \) is that \( h_i = r_f \) for all \( i \), where all individuals compare their investment with bank saving.
3. Asset pricing with both regret aversion and risk aversion

In the previous section, the modified utility function excludes risk aversion. To see this, consider the case of \( h_i = l_i \), where the individual neither has regret nor has rejoice. In this case, the individual’s modified utility function reduces to \( V_i(W_{t,1}) = W_{t,1} \), which is the utility function of a risk neutral agent.

In this section, we extend the analysis to the case where individuals also have risk aversion. To do so, we assume that individuals have the following modified utility function:

\[
V_i(W_{t,1}) = u_i(W_{t,1}) + f_i(W_{t,1} - H_i)
\]

where \( u_i' > 0 \), \( u_i'' < 0 \), and \( E[|u_i''(W_{t,0}(1+r_{ip})|)] < \infty \). Here, similar to the utility function in a standard asset pricing model, \( u_i(\cdot) \) is increasing and concave. The condition \( E[|u_i''(W_{t,0}(1+r_{ip})|)] < \infty \) ensues that the Stein’s Lamma can be applied. If the individual has no regret and rejoice, say, in the case of \( h_i = l_i \), then, the modified utility function \( V_i(W_{t,1}) \) coincides with the utility function of a risk averse agent. The assumptions about \( u_i(\cdot) \) and \( f_i(\cdot) \) are the same as those in the previous section. Therefore, in the modified utility function in equation (13), the concavity of \( u_i(\cdot) \) reveals risk aversion while the concavity of \( f_i(\cdot) \) reveals regret aversion.

Each individual maximizes the expectation of his modified utility. Because both \( u(\cdot) \) and \( f(\cdot) \) are increasing and concave, an equilibrium can be defined as a strategic profile \( \{w_{ij}\} \) that satisfies the first order condition as well as the market clearing condition. By the first order condition and the Stein’s Lemma, we have

\[
E_r f_i(W_{t,0}(1+r_{ip})), E_r u_i(W_{t,0}(1+r_{ip})), E_r h_i, E_r r_j
\]

where \( A_i \equiv \frac{E[u_i''(W_{t,0})]+E[f_i''(W_{t,0}(r_{ip}-h_i))]}{E[u_i''(W_{t,0})]+E[f_i''(W_{t,0}(r_{ip}-h_i))]} \)

From the definition of \( A_i \), we can see that \( 1/A_i \) is similar to absolute risk aversion, only it combines both regret aversion and risk aversion. In this sense, we can call \( 1/A_i \) a measure of the “absolute aversion to risk and regret”. The term \( B_i \) reveals the relative importance of regret aversion. Note that \( B_i = 1 \) if \( u_i(\cdot) \) is linear, in which case the individual has regret aversion but no risk aversion; in contrast, \( B_i < 1 \) if \( u_i(\cdot) \) is strictly concave, in which case the individual has both risk aversion too. Given \( u_i(\cdot) \), the larger is regret aversion, the larger is \( B_i \). Obviously, \( A_i > 0 \) and \( 0 < B_i < 1 \).

Aggregating above equation across individuals, we have

\[
E[r_j] - r_f = \zeta \text{cov}(r_m - h_m, r_j)
\]

where \( h_m \equiv \sum_{i=1}^{M_0} w_{i0} B_i h_i \) and \( \zeta \equiv M_0 (\sum_{i=1}^{M_0} A_i)^{-1} \).

Here, \( h_m \) is the weighted average of \( h_i's \) with \( \{w_{i0}/M_0 \} \) being the weights. \( \zeta \) aggregates \( 1/A_i \) across individuals and multiplies it with the total wealth \( M_0 \). Because \( 1/A_i \) reveals “absolute aversion to risk and regret,” we can call \( \zeta \) the “market-wide relative aversion to risk and regret.” Because \( \zeta > 0 \), equation (15) implies that for an asset with positive covariance \( \text{cov}(r_m - h_m, r_j) \), the larger is \( \zeta \), the higher is the excess return on this asset.

In equilibrium, the market is cleared and the market portfolio satisfies equation (15). As shown in the appendix, a signal beta pricing formula can be derived.

Proposition 4. If \( \text{var}(r_m - h_m) > 0 \), then,

\[
E[r_j] - r_f = \text{cov}(r_j, r_m - h_m) \frac{E[r_m] - E[h_m] - r_j (1 - \zeta)}, \]

where \( \beta_{fg} \equiv \frac{\text{cov}(r_j, r_m - h_m)}{\text{var}(r_m - h_m)} \) and \( \xi \equiv \sum_{i=1}^{M_0} w_{i0} B_i \). Moreover,

\[
\frac{E[r_m] - E[h_m] - r_j (1 - \zeta)}{r_f} > 0.
\]
Proposition 4 states that a security’s excess return is increasing to its regret beta. In equation (16), $\beta_{ij}$ is the single beta that determines the security’s excess return. The term $-\gamma(1 - \xi)$ enters equation (16) because of a technical reason. Note that when $l_i$’s are averaged into $r_m$, the weights are $\left\{\frac{W_{i0}}{M_0}\right\}$; however, when $h_i$’s are averaged into $h_m$, the weights are $\left\{\frac{W_{i0}}{M_0}B_i\right\}$. Because $\sum_{i=1}^l \frac{W_{i0}}{M_0}B_i = 1$ while $\sum_{i=1}^l \frac{W_{i0}}{M_0} = 1$, the term $-\gamma(1 - \xi)$ is needed to adjust for this difference. If we assume $u_i'' = 0$ for all $i$, then, $B_i = 1$ and $\sum_{i=1}^l \frac{W_{i0}}{M_0}B_i = 1$; if so, $-\gamma(1 - \xi) = 0$ holds and the equation (16) is the same as equation (10) in Proposition 1.

Finally, to show explicitly the difference between our model and CAPM, we derive a two beta formula from equation (16).

**Proposition 5.** If $\text{var}(r_m - h_m) > 0$, then,
\begin{equation}
E[\tilde{r}_j] - \gamma = \beta_{jm}(E[r_m] - \gamma) + \hat{\beta}_{jm}(E[h_m] - \xi \gamma)
\end{equation}
where $\hat{h}_m = h_m - \beta_{hm}(r_m - \gamma)$, $\hat{r}_j = r_j - \beta_{jm}(r_m - \gamma)$, and $\hat{\beta}_{jm} = \frac{\text{cov}(\tilde{r}_j, \hat{h}_m)}{\text{var}(h_m)}$. Moreover,
\begin{equation}
E[\hat{h}_m] - \xi \gamma < 0.
\end{equation}

Equation (18) contains a market beta $\beta_{jm}$ and a market-adjusted regret beta $\hat{\beta}_{jm}$. Here, the average countfactual $h_m$ is adjusted to $\hat{h}_m$ by market risk, and return $\gamma$ is adjusted to $\hat{\gamma}$. After excluding market risk, $\hat{\beta}_{jm}$ measures how a security correlates with the average countfactual. Because $E[\hat{h}_m] - \xi \gamma < 0$, the larger is $\hat{\beta}_{jm}$, the lower is the security’s excess return. The intuition of this result is obvious: if a security tends to commove with the average countfactual, it will cause few regret on average; as a result, it is preferred by investors and the price goes up in equilibrium.

Comparing equation (18) with CAPM, it is clear that the term related with $\hat{\beta}_{jm}$ is missing in CAPM. In other words, if we ignore regret aversion and still apply CAPM to the market, abnormal returns will be observed. Moreover, it is easy to see that Proposition 5 holds even when only a small number of individuals are regret averse. This implication of the model is addressed bellow as a remark.

**Remark.** If there are regret averse individuals in the market, CAPM systematically overvalues (undervalues) the expected return of a security that has positive (negative) correlation with market-wide average countfactual after adjusted for market risk.

In the real world, both the proportion of regret averse individuals and their countfactuals may change over time. By analyzing a dynamic model of regret and asset pricing, more testable implications are expected to be obtained. This, however, is left as a future research task.

4. Conclusion

This paper examines the effect of regret on asset pricing in a model where each individual compares the return on his chosen portfolio with a countfactual, which is the return on an unchosen portfolio. In equilibrium, a single beta asset pricing formula is derived where an asset’s expect return of return is increasing to its beta with respect to the difference between the market return and the market-wide average countfactual. This result not only holds in the case where individuals are regret averse but risk neutral, but also holds when individuals are both regret averse and risk averse. The model in the present paper implies that in an asset market where some individuals
regret averse, the market return includes a premium for regret aversion. Moreover, if CAPM is applied to such a market, assets will be systemically mispriced: assets with positive (negative) correlations to the market-wide average counterfactual, calculated after adjusting for market risk, are undervalued (overvalued).

The present paper provides the first asset pricing model under regret aversion. By introducing regret into asset pricing theory, the foundation of modern finance theory, this paper contributes both to the literature on asset pricing and to the literature of Regret Theory.

Reference