

## **Time-varying Equity Risk Premium over Long-run Economic Cycles**<sup>□</sup>

– Empirical evidence of Benartzi-Thaler’s behavioral hypothesis –

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### **Abstract**

This paper proposes a unique way to estimate time-varying equity risk premium (ERP) and equity duration in US and Japan stock markets from 1956 to 2015. We have empirically identified mean-reverting cycles of ERP, responding to booms and busts in financial markets. This observation is consistent with behavioral hypothesis proposed by Benartzi and Thaler (1995) that level of ERP is inversely related to time horizon due to investors’ myopic loss aversion.

JEL: G12, G41

Key Words: Equity Risk Premium, Duration, Myopic Loss Aversion

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## 1. Brief Review of Literature

While there have been innumerable amount of research about ERP estimation both by academics and practitioners, we hereby review only a few critical issues in the past literature. In general, ERP is defined as “forward-looking (ex-ante) excess return over risk-free rate that equity investors demand as compensation for taking extra risk”. Although a risk-free rate is readily observable, ERP is an invisible additional return to be added on the risk-free rate. Some of commonly used ways to estimate ERP are;

**Historical ERP:** Assuming “history repeats itself in the long-run”, a naïve way to estimate ERP is to look back historical performance data over decades, calculate equity excess return over government bonds (risk-free asset) and use the ex-post mean return for an unbiased estimate for ex-ante ERP. It is often criticized as if it is driving a car by looking only at rear mirror.

**Valuation-implied ERP forecast :** Valuation measures such as dividend-price (D/P) ratio and earnings-price ratio (E/P) might be used as a proxy for forward-looking ERP. As long as current stock prices are at equilibrium, they imply ERP demanded by aggregate investors.

**Consensus ERP estimate by investor survey :** A direct way to know what ERP is currently demanded by investors is to simply ask them by a survey. Such surveys were conducted and results were published from time to time. As these surveys were not conducted every month or quarter, it is impossible to keep track of ERP variation over time, especially over decade.

**Supply-side ERP by corporate fundamental :** Instead of using highly volatile equity market returns, recent studies proposed “supply-side approach” by using fundamental data such as ROE and payout ratio. Although supply-side approach might be better way to measure an economy’s real capability of providing equity return, it is not in accordance with theoretical definition of ERP as “demand-side” concept.

None of those approaches satisfies our intention to detect time variation of ERP over long-run economic cycles in U.S. and Japan. We must explore alternative ways to estimate time-varying ERP.

## 2. An Estimation Model: Equity as Quasi-Perpetual Bond

This paper proposes a unique method to converting observed returns to forward-looking ERP implied in markets at each time over long-term history in U.S. and Japan. The basic idea is summarized in three points; (i) As equity can be regarded as quasi-perpetual bond because both do not have maturity, a similar relation should hold among yield changes, durations, and price changes (return variation); (ii) Residual returns over or under the historical mean return reflects time-variation of ERP, having (by definition) mean reverting tendency; (iii) Using identified

relation in (i), we can convert observed returns to estimate ERP at each point in time.

Equity excess return in month  $t$ ,  $X_t$ , is calculated by subtracting bond income return,  $IR_t$ , from equity total return,  $TR_t$ . For the entire sample period,  $X_t$  has a historical mean,  $\mu$ . Observed monthly excess return  $X_t$  is composed of mean  $\mu$  and residual,  $\varepsilon_t$ .

$$x_t = TR_t - IR_t = \mu + \varepsilon_t, \quad (1)$$

Annualized historical risk premium  $\mu_A$  is  $\mu$  (monthly mean) multiplied by 12. While summation of all residuals  $\varepsilon_t$  (deviation from the mean) results in zero by definition, time variation of ERP must be reflected in this residual return.

But variation of ERP may not be the single source of residual returns. Just as changes of long-term interest rate negatively affect bond price returns, part of residual return may be similarly affected by interest rates. In order to extract “pure residual return” excluding interest rate changes, we run a regression (2) by using monthly change of interest rate  $\Delta r_{f,t}$  as independent variable to explain the monthly return  $x_t$ .

$$x_t = a + b \cdot \Delta r_{f,t} + e_t, \quad (2)$$

The constant term  $a$  in equation (2) corresponds to  $\mu$  in equation (1). The second term  $b \cdot \Delta r_{f,t}$  is the impact of interest rate, and the third term,  $e_t$ , is the “pure residual excess return” reflecting ERP changes. Again, sum of all  $e_t$ , is zero, and  $e_t$ , is supposedly showing mean reverting tendency. We use  $e_t$  as the key estimator for time-varying ERP.

Let  $\lambda_t$  denote a level of ERP at the end of month  $t$ . For month  $t$ , a small change of ERP,  $\Delta\lambda_t$  ( $=\lambda_t - \lambda_{t-1}$ ) and residual return are inversely related. For each month  $t$ , we construct an index indicating monthly change of ERP,  $(1 + \Delta\lambda_t)$ , from an inverse of incremental return from residual,  $(1 + e_t)$ .

$$(1 + \Delta\lambda_t) = (1 + e_t)^{-1}, \quad (3)$$

Setting the starting value of index as 1.00 at the beginning of period (December 1955 in this study), the index value  $I_m$  for the  $m$ 'th month can be calculated as cumulative sum of monthly  $\Delta\lambda_t$  as formula (4).

$$I_m = 1 + \sum_{t=1}^m \Delta\lambda_t, \quad (4)$$

The index value  $I_m$  is relative deviation (higher or lower) from the historical mean of ERP,  $\lambda_t$ . Where  $I^*$  represents the mean value of  $I_m$  that corresponds to the annualized historical excess return or ERP  $\mu_A$ , an estimated value of ERP for month  $m$ ,  $\lambda_m$ , is obtained simply by equation (5).

$$\lambda_m = \mu_A \times I_m / I^*, \quad (5)$$

### 3. Data: Historical Return in U.S. and Japan over 60 years

To estimate time-varying ERP based on empirical data, this paper uses monthly return data in U.S. and Japanese financial markets over 60 years from January 1956 to December 2015. Equity

market total return series are Morningstar’s SBBI Large cap equity for US, which is virtually identical to S&P500, and TOPIX with dividend for Japan. Risk-free rate series are long-term government bond income returns for both countries. Bond yields are used when we estimate base-rate for forward looking ERP. Inflation rate are consumer price index (CPI) for both countries, and are used to calculate implied real rate adjusted for inflation. Using those return series, we derive equity excess return (EXR), commonly known as “historical equity premium”, by subtracting bond income return from equity total return for each month. A summary statistics of those data series are shown in **Table 1**.

**Table 1: Summary Statistics of Historical Returns from 1956 to 2015**

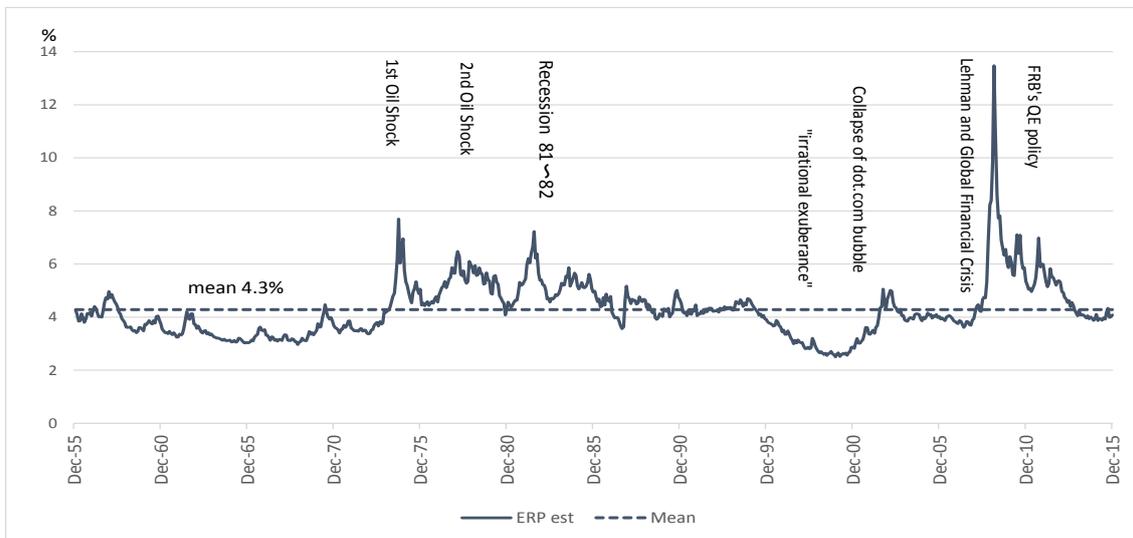
		( Annual % )						
		Arithmetic Mean			Standard Deviation			
		Time Period	First	Second	All	First	Second	All
			1956-1980	1981-2015	1956-2015	1956-1980	1981-2015	1956-2015
U.S.	①	Inflation	4.7	2.9	3.7	1.2	1.2	1.2
	②	Yield on Long-term Bond	5.9	6.7	6.4	2.1	2.9	2.7
	③=②-①	Real Yield	1.2	3.8	2.7	1.8	2.1	2.3
	④	Bond Income Return	5.8	6.6	6.3	0.8	0.8	0.7
	⑤	Equity Total Return	9.1	11.6	10.6	14.0	15.0	14.6
	⑥=⑤-④	Excess Return (EXR)	3.3	5.0	4.3	13.9	15.0	14.6
	⑦=⑥-①	Equity Real Return	4.4	8.7	6.9	14.1	15.1	14.7
Japan	①	Inflation	6.2	0.8	3.1	1.5	1.5	2.4
	②	Yield on Long-term Bond	7.0	3.3	4.9	0.9	2.5	2.7
	③=②-①	Real Yield	0.8	2.5	1.8	4.3	1.9	3.2
	④	Bond Income Return	6.6	3.4	4.7	0.7	0.7	0.7
	⑤	Equity Total Return	15.7	6.3	10.2	15.9	18.8	17.7
	⑥=⑤-④	Excess Return (EXR)	9.1	2.9	5.5	16.1	18.8	17.7
	⑦=⑥-①	Equity Real Return	9.5	5.5	7.1	16.6	18.9	17.9

Source: Morningstar, Ibbotson Associates Japan.

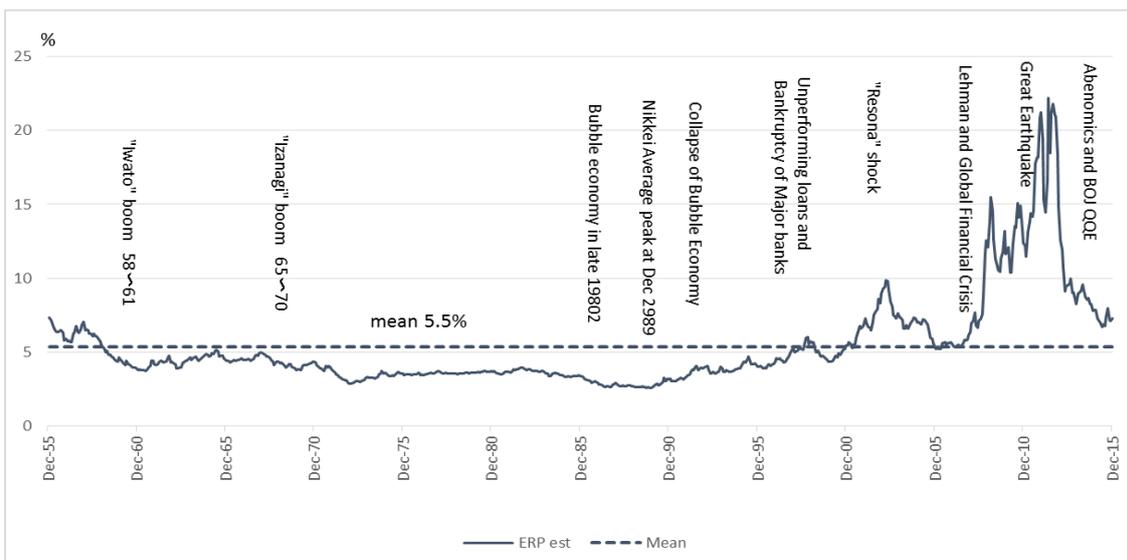
#### 4. Estimation Results: Equity Risk Premium, Duration and Real Expected Return

##### 4.1. Equity Risk Premium

Using monthly return data series in Section 4 to the estimation model described in Section 3, time-varying ERP path is estimated for U.S. in **Figure 1** and for Japan in **Figure 2**. As we anticipated, ERP has mean-reverting tendency in the long-run but it also experience occasional “spikes” from time to time. Such ERP spikes are observed around unexpected financial crisis due to external shocks (e.g. oil crisis) or endogenous shocks (e.g. Lehman collapse). Conversely, declining ERP is observed when economy is on up-trend (e.g. “irrational exuberance” during 90’s in U.S. and “Abenomics” in recent Japan). When such boom extends to extreme euphoria, ERP comes down below 3%. This must be a sign of “dangerous zone” where equity is excessively overvalued and would eventually go bust.



**Figure 1: ERP time-variation in U.S. 1956-2015**



**Figure 2: ERP time-variation in Japan 1956-2015**

**4.2. Equity Duration**

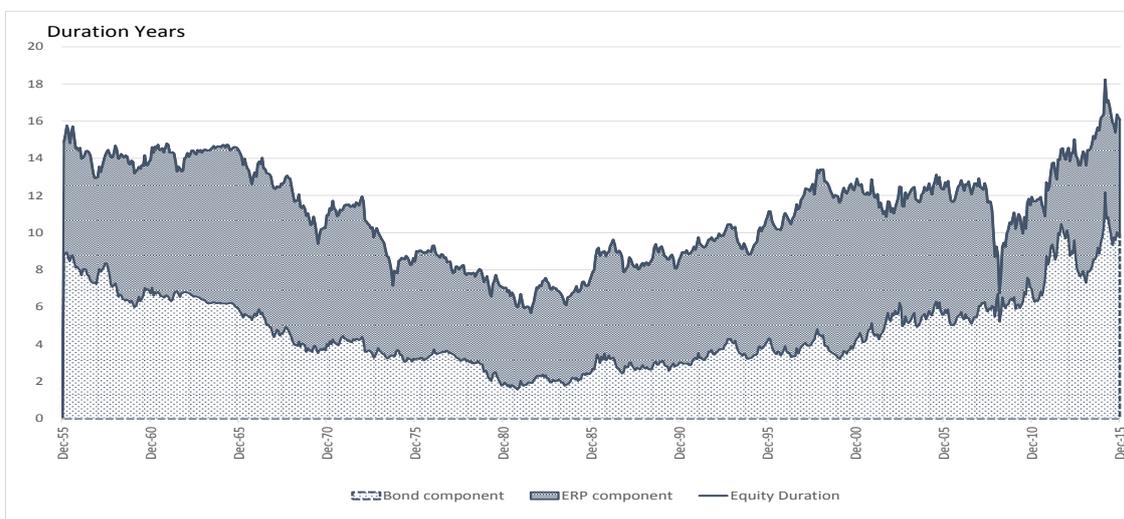
The concept and formula of equity duration was theoretically presented by Boquist et.al. (1975) more than 40 years ago, although they did not measure it by empirical data.

“Nominal” equity expected return is “yield” of equity as quasi-perpetual bond. Where  $D_t$  is duration of perpetual bond at month  $t$ , and  $y_t$  is its yield, duration of equity is then calculated by perpetual bond duration formula in equation [6] below.

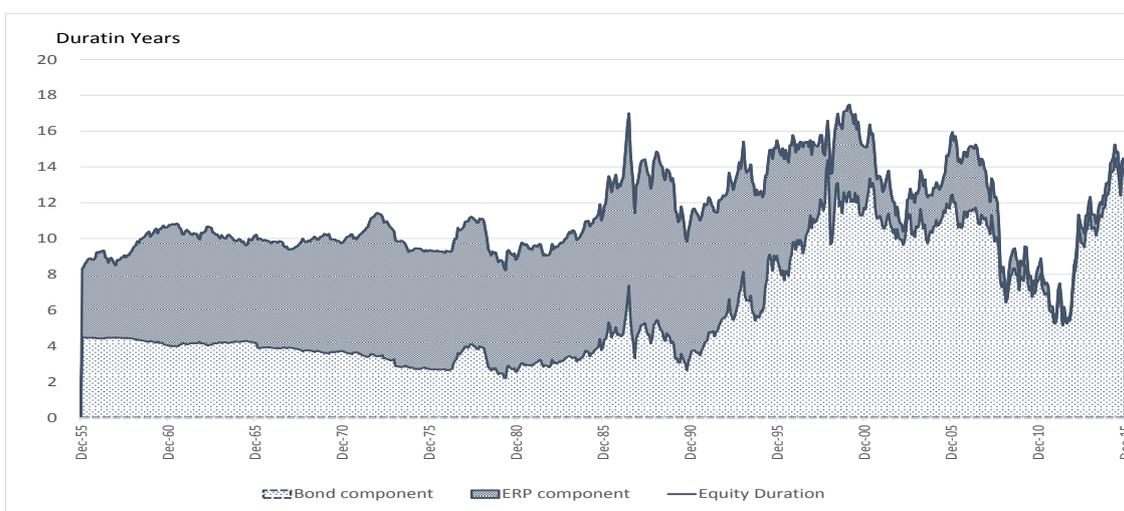
$$D_t = (1+y_t)/y_t \dots [6]$$

Since equity expected return is composed by bond portion and ERP, duration can be separately calculated for each part and weighted by respective components. **Figure 3** is estimated equity duration over time for U.S. market, and **Figure 4** for that of Japan. In both markets, equity duration have moved within a range between 6 years to 16 years. The primary forces driving equity duration is bond components in both markets. When

In general, duration is anticipated time horizon that people are willing to commit their investments. When they are optimistic for future, ERP becomes lower and duration becomes longer. When they are pessimistic, ERP is higher and duration is shorter. This relationship of investors' sentiment with time horizon and ERP is consistent with theoretical argument on myopic loss aversion and ERP by Benartzi and Thaler (1995).



**Figure 3: Equity Duration in U.S. 1956-2015**



**Figure 4: Equity Duration in Japan 1956-2015**

## 5. Empirical Evidence of Myopic Loss Aversion Hypothesis

In previous section, we have empirically quantified ERP and duration over 60 years in U.S. and Japan. How are ERP and duration related? Our observations indicate that ERP level tend to be larger and ERP duration tend to be shorter when economy and financial market were depressed. This intuitively makes sense because investors demand higher ERP and lose patience to commit their investments for longer period into future when economic conditions are perceived as more uncertain. In short, they become more “myopic” and “risk averse” under such risky environment.

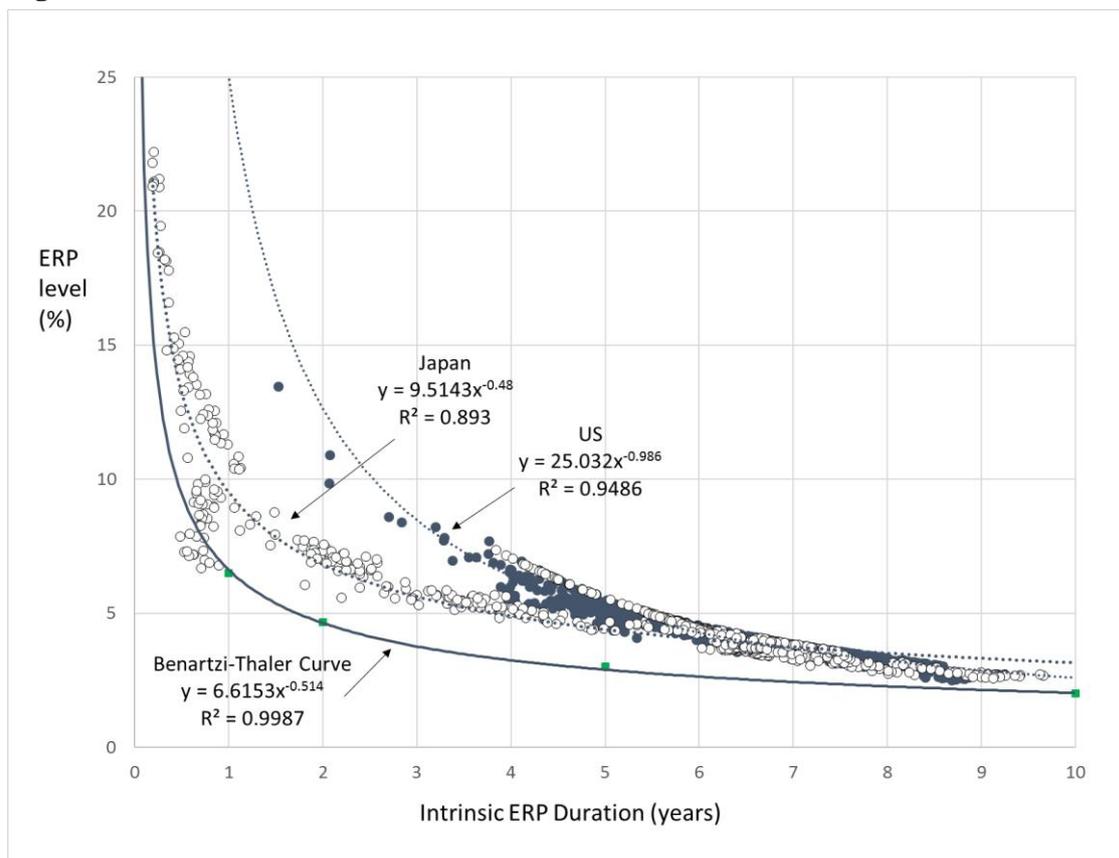
Combining those two behavioral characteristics of investors, Benartzi and Thaler (1995) proposed “myopic loss aversion” hypothesis that explains the risk premium puzzle ---why historically observed ERP (e.g. 6.5%) was much higher than ERP (e.g. 1%) theoretically predicted by Mehra and Prescott (1985). According to Benartzi and Thaler’s calculation, investors with “myopic loss aversion” must have shorter time horizon for reviewing their investment performance frequently and demand higher ERP. Let us call this declining ERP over investment time horizon as “Benartzi-Thaler Curve” (B-T Curve).

Based on our previous estimates at 720 months, **Figure 5** plots intrinsic ERP duration contribution on horizontal axis and ERP level on vertical axis. Black dots (●) represent observations for U.S., and white dots (○) represent those for Japan. To contrast with those empirical observations, B-T Curve (solid line curve) is shown in the same space. When we apply power function to fit each of three data set, we obtained results below. Where  $y$  is ERP level, and  $x$  is ERP duration;

U.S. observations:	$y=25.03x^{-0.986}$	$R^2=0.949$
Japan observations:	$y=9.51x^{-0.48}$	$R^2=0.893$
B-T Curve:	$y=6.61x^{-0.514}$	$R^2=0.999$

These results roughly serve as empirical confirmation of Benartzi and Thaler’s prediction. First, observed curves both in U.S. and Japan have similar shape as B-T Curve. Both markets have similar ERP level where duration is beyond 5 years. Second, however, their empirically estimated ERP levels are slightly higher than their theoretical predictions over all duration. Third, Japan’s empirical curve is about 2 percent higher than and almost parallel with B-T Curve, extending well below one year in duration (horizontal axis) due to deflationary economy during years from 2008 to 2012. Fortunately, U.S. market has not experienced such abnormally extended deflation in the last six decades.

**Figure 5: Duration and ERP**



## 6. Conclusions

This paper proposed a unique way of estimating time-varying equity risk premium by converting historical ex-post data to forward-looking ex-ante data. The key assumption is to regard equity as quasi-perpetual bond. Applying the simple mathematical relation between bond yield, price and duration, time-varying ERP is estimated from historical data.

Monthly ERP estimates have reflected economic events – booms and busts --- for U.S. and Japanese markets over 60 years. During boom periods, ERP becomes lower and duration becomes longer. In economic crisis, they move the other ways. This empirical observation is consistent with Benartzi and Thaler’s theoretical prediction based on myopic loss aversion. This empirical results may be also useful for practical investment decisions such as dynamic asset allocation for medium terms.

**Appendix: Impact of Bond Yield Change on Equity Excess Return**

	$x_t = a + b \cdot \Delta r_{f,t} + e_t$							
	$x_t$ : Monthly excess return of equity total return over government bond income return							
	$\Delta r_{f,t}$ : Monthly change of government bond yield							
	Period	N of Obs	a	t-stat	b	t-stat	R <sup>2</sup>	
U.S.	1956-1980	300	0.37	1.59	-3.09	-3.05	0.03	
	1980-2015	420	0.38	1.79	-1.74	-2.71	0.01	
	All Period	720	0.36	2.29	-2.09	-3.91	0.02	
Japan	1956-1980	300	0.78	2.93	-1.85	-1.14	0.00	
	1980-2015	420	0.22	0.83	-0.81	-0.75	0.00	
	All Period	720	0.45	2.35	-0.98	-1.10	0.00	

To extract “pure residual return” driven by monthly change of ERP, we run regression that uses  $x_t$  (monthly excess return of equity over bond income return) as the dependent variable and  $\Delta r_{f,t}$  (monthly change of government bond yield) as independent variable. As we have already subtracted bond income return from equity total return to derive  $x_t$ , we expect that impact of bond yield change would be minimal if any. Table above confirms this by showing R<sup>2</sup> close to zero both in U.S. and Japan data, while t-statistics are statistically significant for U.S. market. This indicates that our analytical inference is not materially different whether we use the simple residual return ( $\varepsilon_t$  in Equation [1]) or “pure residual return” ( $e_t$  in Equation [2]).

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