A modified inspector leadership game with psychological factors

Masakazu Fukuzumi[1]
University of Tsukuba, Graduate School of Humanities and Social Sciences

Abstract

We formulate a monitoring model which is a modified inspector leadership game where a principal (an inspector) monitors an effort level chosen by an agent (an inspectee). [2] We introduce psychological factors (a sense of guilty, an impulse to deceive and a reciprocity to kindness) into the modified inspector leadership game and examine impacts of these psychological factors on an error probability that the principal conducts a *costly* investigation into an effort level chosen by the agent *although* the agent chooses a desirable level of the effort for the principal. We show that the agent's sense of guilty reduces the error probability but the agent's impulse to deceive raises the error probability. Finally, we show that however large the error probability is, the agent with reciprocity has an incentive to choose an *undesirable* level of the effort for the principal.

Keywords: inspection game, psychological equilibrium, sense of guilty, impulse to deceive, reciprocity, kindness

1 The model without psychological factors

We consider a dynamic game with two players, P (principal) and A (agent). The game is described in Figure 1.

1. At the first stage of the game, player P chooses a probability $\alpha \in [0, 1]$ that player P monitors player A *ex post* by conducting an investigation that provides player P with correct information on an effort level chosen by player A.

[1] 1-1-1 Ten'no-dai, Ibaraki 305-8571, Japan; fukuzumi@social.tsukuba.ac.jp

[2] Inspector leadership game is a class of *inspection games*. Inspection games are mathematical models of a situation where a player verifies that the opponent player chooses a legal rule. These games are applied to analyses of a safeguard system against *nuclear weapons*, material accountancy systems and auditing. In this paper we modify an inspector leadership game to analyze workers' moral hazard problems. For details of inspection games, see Avenhause, Okada and Zamir (1991), Avenhaus and Okada (1992) and Avenhaus, von Stengel and Zamir (2003). Englemaier (2005) is a survey of behavioral game theoretic models about workers' moral hazard problems.

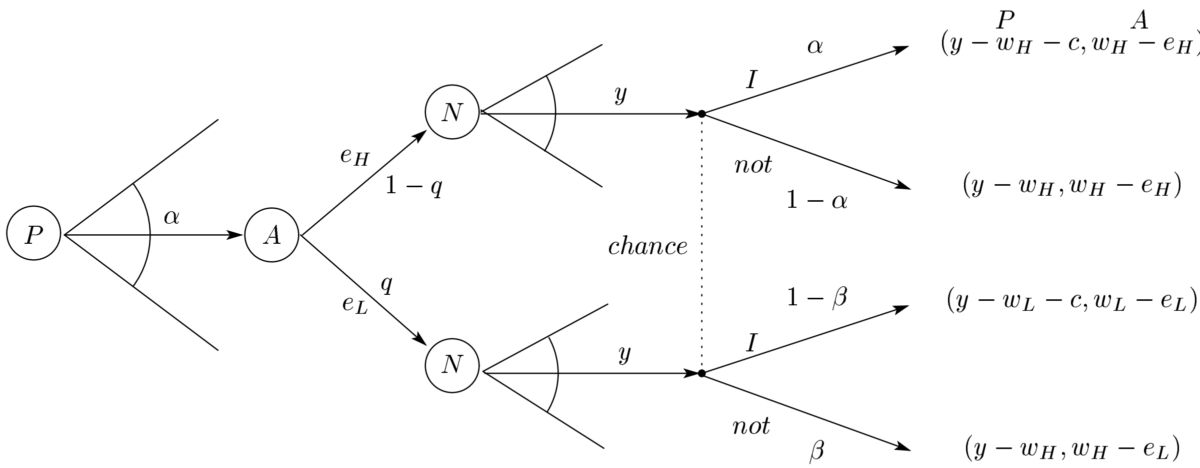

Figure 1: The model without psychological factors.

2. At the second stage of the game, player A given the probability α chooses his effort level e_l from a set $\{e_H, e_L\}$. Each effort level e_l is a nonnegative real number and $e_H > e_L$. We consider a behavior strategy given by the probability $q \in [0, 1]$ for choosing the low effort e_L.

3. At the third stage of the game, Nature picks up output $y \in \mathbf{R}$. If player A has chosen e_l at the previous stage, the corresponding output y is realized according to a cumulative distribution function $F_l(y)$ which has the mean $\mu_l \in [0, \infty)$ where $l = L, H$, and $\mu_H > \mu_L$ and which has an identical variance with each other. Each distribution function F_l $(l = L, H)$ is absolutely continuous and it has an inverse function F_l^{-1}.

4. At the fourth stage, if an output y realized at the previous stage belongs to a set $Z_\alpha \equiv \{ \ y \mid F_H(y) \leq \alpha\}$, player P conducts an investigation for the effort level chosen by player A. It costs a fixed amount of $c > 0$ unit of output for player P to conduct the investigation. After the investigation:

   - if player A has chosen e_H at the second stage of the game, then player P gives fixed wage $w_H \in \mathbf{R}$ to player A,
   - if player A has chosen e_L at the second stage of the game, then player P gives fixed wage $w_L \in \mathbf{R}$ to player A where $w_L < w_H$.

   If an output y realized at the previous stage does not belong to the set $Z_\alpha \equiv \{ \ y \mid F_H(y) \leq \alpha\}$, then player P does not conduct the investigation and gives the fixed wage w_H to player A.

Payoff of each player

Let e_l be the effort level chosen by the agent at the second stage of the game. If the output y realized at the third stage of the game belongs to the set Z_α, each payoff of player P and of player A is given by $y - w_l$ and $w_l - e_l$, respectively. If the output y realized at third stage of the game does *not* belong to the set Z_α, each payoff of player P and of player A is given by $y - w_H$ and $w_H - e_l$, respectively. Let $z_\alpha \equiv F_H^{-1}(\alpha)$. Then the expected payoff of each player in our model is given by

$$\begin{aligned} Eu_A(\alpha, q) &= q\Big[\int_{z_\alpha}^{+\infty}(w_H - e_L)dF_L(y) + \int_{-\infty}^{z_\alpha}(w_L - e_L)dF_L(y)\Big] \\ &+ (1-q)\Big[\int_{z_\alpha}^{+\infty}(w_H - e_H)dF_H(y) + \int_{-\infty}^{z_\alpha}(w_H - e_H)dF_H(y)\Big], \quad (1) \end{aligned}$$

$$\begin{aligned} Eu_P(\alpha, q) &= q\Big[\int_{z_\alpha}^{+\infty}(y - w_H)dF_L(y) + \int_{-\infty}^{z_\alpha}(y - w_L - c)dF_L(y)\Big] \\ &+ (1-q)\Big[\int_{z_\alpha}^{+\infty}(y - w_H)dF_H(y) + \int_{-\infty}^{z_\alpha}(y - w_L - c)dF_H(y)\Big]. \quad (2) \end{aligned}$$

We assume that the market-imposed minimal expected payoff for player A is 0, that is, $w_L - e_L = 0$.

Relationship between hypothesis testing in statistics and our model

The null hypothesis H_0 in our model is that player A chooses the high effort e_H, and the alternative hypothesis H_1 in our model is that player A chooses the low effort e_L. The probability α chosen by player P is that of *the error of the first kind* in hypothesis testing. Namely, the value of α is the probability that the principal conducts the *costly* investigation *althouth* the agent chooses the high effort e_H.

We denote by $\beta \in [0, 1]$ the probability of *the error of the second kind* in hypothesis testing. Namely, the value of β is the probability that the player P does *not* conduct the investigation into the effort level chosen by player A *although* player A chooses the low effort e_L. Moreover, we obtain a function $\beta = 1 - F_L(z_\alpha)$ where $z_\alpha \equiv F_H^{-1}(\alpha)$. The function $\beta(\alpha)$ fulfills $\beta(0) = 1$ and $\beta(1) = 0$.

Assumption 1. The function $\beta(\alpha) \in [0, 1]^{[0,1]}$ is a differentiable, convex, and monotonically decreasing function.[3]

[3] $\beta(\alpha) \in [0, 1]^{[0,1]}$ denotes a function $\beta(\alpha)$ on $[0, 1]$ into $[0, 1]$. In the following, we use similar notations. For example, $G(\alpha) \in \mathbf{R}^{[0,1]}$ denotes a function $G(\alpha)$ on $[0, 1]$ into $\mathbf{R}$.

We obtain a benchmark of this paper.

Theorem 1.1. The subgame perfect equilibrium point (α^*, q^*) of our model without psychological factors is given by a pair of $\alpha^* = \beta^{-1}(1 - \frac{e_H - e_L}{w_H - w_L})$ and $q^* = 0$.

2 The model with a sense of guilty

We introduce a psychological factor, a sense of guilty of player A, into our model. Let $q'' \in [0,1]$ be player A's belief about player P's belief about a behavior strategy $q \in [0,1]$ which is a probability that player A chooses the low effort e_L. We call the belief $q'' \in [0,1]$ the *second order belief* of player A.

Consider a situation where player A chooses the low effort e_L and player P does *not* conduct an investigation into the effort level chosen by player A. In this situation the second order belief q'' of player A is the smaller one, the more player A feels guilty about his choosing the low effort. We add an increasing function $g(q'')$ to player A's payoff of this situation. [4]

Assumption 2. $g \in \mathbf{R}^{\mathrm{R}}$ is a differentiable and monotonically increasing function and fulfills that $g(0) = -k$ and $g(1) = 0$ where $k > 0$.

Namely, the value of $-g(q'')$ captures the strength of the sense of guilty of player A with q''.

In the following of this paper we use an equilibrium concept, *psychological equilibrium*, given by Geanakoplos, Pearce and Stacchetti (1989) and Rabin (1993).

Definition 2.1. A *psychological equilibrium point* of our model with a psychological factor is a triplet $(\alpha^{**}, q^{**}, q'')$ such that
(1) the pair of (α^{**}, q^{**}) is the subgame perfect equilibrium point of our model with a psychological factor and
(2) $q^{**} = q''$. (*consistency*)

After finding player A's best response correspondence, we obtain a following result.

Theorem 2.1. The psychological equilibrium point $(\alpha^{**}, q^{**}, q'')$ of our model with player A's sense of guilty is given by $(\alpha_l, 0, 0)$ where

[4] Dufenberg (2002) proposed a trust game with a sense of guilty.

$\alpha_l = \max\{\beta^{-1}\big(\frac{(w_H - w_L)-(e_H - e_L)}{(w_H - w_L)-k}\big), 0\}$.

Since $\alpha^* = \beta^{-1}(1 - \frac{e_H - e_L}{w_H - w_L})$ and $k > 0$, it turns out that $\alpha_l < \alpha^*$.

3 The model with an impulse to deceive

We introduce a psychological factor, player A's impulse to deceive player P, into our model formulated in Section 1. Consider a situation where player A chooses the low effort e_L and player P does *not* conduct the investigation for the effort level chosen by player A. In this situation the second order belief q'' of player A for choosing the low effort e_L is the smaller one, player A with an *impulse to deceive* player P may feel the more satisfaction with this situation. We add a decreasing function $g_d(q'')$ to player A's payoff of this situation.

Assumption 3. $g_d \in \mathbf{R}^{\mathbf{R}}$ is a differentiable and monotonically decreasing function and fulfills that $g_d(0) = k_d$ and $g_d(1) = 0$ where $k_d > 0$.

After finding player A's best response correspondence, we obtain a following result.

Theorem 3.1. The psychological equilibrium point $(\alpha^{**}, q^{**}, q'')$ of our model with player A's impulse to deceive is given by $(\alpha_m, 0, 0)$ where

$\alpha_m = \beta^{-1}\big(\frac{(w_H - w_L)-(e_H - e_L)}{(w_H - w_L)+k_d}\big)$.

Since $\alpha^* = \beta^{-1}(1 - \frac{e_H - e_L}{w_H - w_L})$ and $k_d > 0$, it turns out that $\alpha^* < \alpha_m$.

4 The model with reciprocity

We introduce player A's reciprocity into our model formulated in Section 1. [5] A formulation $\{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\}$ measures player P's kindness as perceived by player A. A formulation $\{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$ measures how much player A alters player P's payoff with his own behavior strategy q. Namely the product of $\{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\}$ and $\{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$ measures the reciprocity utility of player A.

We assume that the expected payoff $Eu_A^R(\alpha, q, q'')$ of player A with reciprocity is

$Eu_A^R(\alpha, q, q'') = Eu_A(\alpha, q) + \rho_A\{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\} \times \{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$, (1.1)

We get an inequality $\frac{\partial Eu_A^R}{\partial q}(\alpha, 0, 0) > 0$ by simple calculation, so that obtain a following result.

[5] Dufwenberg and Kirchsteiger (2004) and Falk and Fischbacher (2006) propose theories of reciprocity.

Theorem 4.1. Player P (an inspector) can not prevent player A (an inspectee) with reciprocity from choosing the low effort e_L in our inspector leadership game.

Note that Theorem 3.1 shows that player P (an inspector) can prevent player A (an inspectee) with an impulse to deceive from choosing the low effort e_L. The reciprocal agent feels unkindness of the principal in the inspector leadership game.

References

1. Avenhaus, R., and A. Okada (1992) "Statistical Criteria for Sequential Inspector-Leadership Games," Journal of The Operations Research Society of Japan **35**: 134–151.

2. Avenhaus, R., A. Okada, and S. Zamir (1991) "Inspector Leadership with Incomplete Information," in R. Selten (eds.) *Game Equilibrium Models* **4**. Springer Berlin: 319–361

3. Avenhaus, R., B. von Stengel, and S. Zamir (2002) "Inspection Games," in R.-J. Aumann and S. Hart (eds.) *Handbook of Game Theory with Economic Applications* **3**. Elsevier Science B.V: 1948–1987.

4. Dufwenberg, M., and G. Kirchsteiger (2004) "A Theory of Sequential Reciprocity," Games and Economic Behavior **47**: 268–298.

5. Dufwenberg, M. (2002) "Marital Investments, Time Consistency and Emotions," Journal of Economic Behavior and Organization **48**: 57–69.

6. Englmaier, F. (2005) "Moral Hazard, Contracts and Social Preference," in B. Agarwal and A. Vercelli (eds.) *Psychology, Rationality and Economic Behavior.* Palgrave Macmillan H.B.H: 125–139

7. Falk, A., and U. Fischbacher (2006) "A Theory of Reciprocity," Games and Economic Behavior **54**: 293–315.

8. Geanakoplos, J., D. Pearce, and E. Stacchetti (1989) "Psychological Games and Sequential Rationality," Games and Economic Behavior **1**: 60–79.

9. Rabin, M. (1993) "Incorporating Fairness into Game Theory and Economics," American Economic Review **83**: 1281–1302.