Investor Confidence, Short-Sales Constraints and the Behavior of Security Prices[1]

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Abstract

Investor confidence affects financial markets. Information, noise, market frictions cause investor confidence to influence security prices, leading to a price different from the rational expectations value. This paper presents a simple theoretical model of asset prices where investor confidence is allowed to differ across traders. We examine the resulting equilibrium, compare it to the fully rational case and characterize conditions necessary for the price to diverge from its fundamental value. Furthermore, in a market with short-sales constraints, asset prices are found to behave asymmetrically: short-run returns display reversal after good news, but momentum after bad news.

Keywords: Behavioral Finance, Overconfidence, Short-Sales Constraints, Momentum, Reversal

1. Introduction

Behavioral finance has undergone rapid development in recent years. A number of new asset pricing models have been proposed, most of them highly stylized, yet elegant and insightful. The assertion of behavioral finance is that security prices and expected returns are based both on risk and investor misvaluation, leading to mispricing. Equilibrium prices thus reflect a weighted average of both rational and irrational traders. One of the most widely discussed behavioral phenomena concerned with financial asset markets is that of *overconfidence*, i.e. a tendency for people to put too much weight in their own judgments, to believe their own point of view to be more accurate than it

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actually is when considered objectively. In this article, interaction between overconfident and underconfident traders in a financial market setting is analyzed. While overconfidence is a well-documented pervasive pattern of human behavior, underconfidence seems to be a less salient phenomenon. However, it may become more understandable why underconfidence, and generally differing confidence among different traders, is employed in our model, if we observe its connection with noise. In his influential essay, Black (1986) remarks that "people sometimes trade on noise as if it were information". He further asserts that "people who trade on noise are willing to trade even though from an objective point of view they would be better off not trading". In today's world of information flooding, distinguishing noise from valuable information is an extremely difficult task. This observation motivates the approach in this paper based on investor psychology rather than the lately more common practice of asymmetric information-based models. The investors in possession of a piece of information can never be sure that they are actually trading on information rather than noise. It is possible the information has already been reflected in prices – trading on that sort of information would be just like trading on noise. It is therefore plausible to say that investors who are uncertain as to the quality and relevance of their information are underconfident. The effect of short-sales constraints is also examined in a behavioral finance context.

2. Investor Confidence and Short-Sales Constraints

This section presents the formal analysis of asset prices with emphasis on investor confidence, short-sales constraints and the interplay of these two factors.

2.1. Two Types of Traders with Opposing Biases: Security Price Behavior

Our model is basically similar to one of the staple models of behavioral finance, namely that of Daniel, Hirshleifer, and Subrahmanyam (1998). We begin the formal analysis with a simple model of asset prices with two types of boundedly rational investors present in the market. The first type overestimates the accuracy of their information whilst the second type of investors underestimates the precision of theirs. Let us start the analysis with the characterization of the model structure. There is one risk-free asset with constant payoff equal to unity and one risky stock with net supply 0. There are three dates: $t=0$, $t=1$, $t=2$. At $t=2$, the stock pays a terminal dividend equal to F. It is assumed to be normally distributed according to $F \sim N(0, \sigma_F^2)$. At $t=1$, all traders receive a noisy signal about the risky asset's intrinsic value:

$$s_1 = F + \varepsilon \ , \tag{1}$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Thus, the signal precision is given by the reciprocal of its variance, $1/\sigma_\varepsilon^2$. All random variables are independent and normally distributed. At $t=0$, the price is simply its prior mean, $P_0=0$. Investors misperceive the *precision* of the information they receive; such a bias is modeled in the following way: an investor's k's belief about the distribution of his signal is $s_k=F+B_k\varepsilon$, where $k=c,d$. Overconfident investors, indicated with the subscript "c" believe their signal is more accurate than it actually is, i.e. $0<B_c<1$; thus they believe its distribution to be "too tight". On the other hand, underconfident investors, denoted by "d" believe the distribution of their signal to be "too loose"[3], i.e. $B_d \geq 1$. A possible justification for this structure is a situation where different investors get their information – which is objectively the same – from different sources: one reliable and the other questionable, or "untrustworthy". Then one group will believe the precision of their signal to be more accurate than it truly is, whilst the other group will think their signal contains too much noise. Such an arrangement results effectively in a structure that is possible to be characterized by overconfidence and underconfidence. Let the subset of investor population consisting of overconfident traders be λ, so that the number of underconfident traders is $1-\lambda$, where $0<\lambda<1$.

All investors have CARA (Constant Absolute Risk Aversion) utility functions with equal risk tolerance coefficient γ:

$$E[U(W_k)] = E[-\exp(-\frac{W_k}{\gamma})] \tag{2}$$

With normal distributions, this implies a mean-variance utility function. In a multiperiod model, wealth depends on investor decisions in all the periods. Unfortunately, the general solution for such a problem is quite complex; we therefore focus on "*myopic*" behavior: traders are assumed to focus only on the immediate period, and so decisions are independent across periods. This effectively ignores any interperiod linkages but does allow the problem to be analyzed tractably. The wealth at the final date for each trader is the sum of the initial wealth W_0 and the gain derived from the two types of assets. Since the payoff of the risk-free asset is always one, it follows that for trader k ($k=c,d$): $W_k=W_0+D_k(F-P)$, where D_k is trader k's demand for the risky asset and P is its price. The trader k's maximization problem is therefore:

$$\max_{D_k} \quad \mathrm{E}_k[W_k|s] - \frac{Var[W_k|s]}{2\gamma} \quad \text{s.t.} \quad W_k = W_0 + D_k(F-P) \tag{3}$$

By standard properties of normal variables it follows that:

[3] The case of $B_d=1$ is allowed for ease of analysis in the special case of the overconfident investors interacting with perfectly rational investors.

$$\mathrm{E}_k[F|s] = \mathrm{E}[F] + \frac{Cov[F,s]}{Var[s]}(s - \mathrm{E}[s]), \quad Var_k[F|s] = Var[F] - \frac{Cov[F,s]Cov[s,F]}{Var[s]},$$

which in the model considered here becomes:

$$\mathrm{E}_k[F|s] = \frac{\sigma_F^2}{\sigma_F^2 + B_k^2\sigma_\varepsilon^2}(F+\varepsilon) \tag{4}$$

$$Var_k[F|s] = \frac{\sigma_F^2 B_k^2 \sigma_\varepsilon^2}{\sigma_F^2 + B_k^2\sigma_\varepsilon^2}, \tag{5}$$

where B_k=B_c for overconfident traders, and B_k=B_d for underconfident traders. Furthermore, solving the above maximization problem yields the following demand functions:

$$D_k = \frac{\gamma[\alpha_k(F+\varepsilon) - P]}{\beta_k}, \tag{6}$$

where $\alpha_k = \frac{\sigma_F^2}{\sigma_F^2 + B_k^2\sigma_\varepsilon^2}$ and $\beta_k = \frac{\sigma_F^2 B_k^2 \sigma_\varepsilon^2}{\sigma_F^2 + B_k^2\sigma_\varepsilon^2}$. It can be seen that α_c>α_d and β_c<β_d, resulting in $|D_c|$>$|D_d|$. To put it another way, overconfident traders' higher conditional mean and lower conditional variance result in them taking larger positions in the risky stock. The assumption of differing signal distributions leads to an equilibrium, which is not a rational expectations equilibrium. The model's equilibrium is thus characterized by the investors' optimal demands as given in (6) and by the market clearing condition equating total demand with total supply:

$$\lambda D_c + (1-\lambda)D_d = 0. \tag{7}$$

Substituting appropriate demand functions to the last equation results in the equilibrium price being equal to:

$$P_1 = \frac{\lambda\alpha_c\beta_d + (1-\lambda)\alpha_d\beta_c}{\lambda\beta_d + (1-\lambda)\beta_c}(F+\varepsilon). \tag{8}$$

Having obtained the equilibrium price, we now turn our attention to the analysis of how this price relates to one that would be obtained under rational expectations; in doing so, the emphasis is applied at the issue of whether the above price can be equal to the "rational" price. In other words, the goal is to check if differing biases might cancel each other out leading to prices being close or equal to those in case of only rational investors being present. To examine the possibility of the price being equal to its corresponding rational expectations value, let us first observe that this price would be equal to

$$P_1^r = \alpha_r (F+\varepsilon) = \frac{\sigma_F^2}{\sigma_F^2 + \sigma_\varepsilon^2}(F+\varepsilon)\,, \tag{9}$$

where "r" indicates the "rational" price. This would be the case if all the investors correctly estimated the precision of their signals, that is if $B_c=B_d=1$. Equating the last two formulas, $P=P_r$, yields after some algebra a condition necessary for the two biases to cancel each other out, leading to a rational equilibrium asset price:

$$1 = \lambda \frac{1}{B_c} + (1-\lambda)\frac{1}{B_d}\,. \tag{10}$$

The above equation has some intuitive properties. Whether the price can attain its rational expectations value, will depend on the interplay of the three parameters: λ, B_c, and B_d. It can be seen that it is only in a special case that the price can attain its rational value. If the first term on the right-hand side of equation (10) prevails, overconfident investors' dominance will show in the price being above its rational level; the opposite happens when the underconfident investors dominate the market. The asset's price depends on the extent of investor confidence and the relative fractions of the two trader types in the whole population. Concentrating on the overconfident fraction of the trader population, it follows that the price cannot be rational if $\lambda>B_c$. To see this, observe that the above condition can be rewritten as

$$B_d = \frac{B_c(1-\lambda)}{B_c-\lambda}\,, \tag{11}$$

which cannot be fulfilled in this case since $B_d \geq 1$. Hence, we have

Proposition 1. *The price of a risky asset in a market populated by overconfident and underconfident traders cannot be equal to its rational counterpart if the fraction of overconfident traders is larger than the overconfident traders' bias, that is, if* $B_c<\lambda$.

2.2. Short-Sales Constraints

We now turn to the next part of the analysis, where the presence of short-sales constraints is assumed. Assume the $t=1$ signal was positive. Whether fully rational or underconfident in the precision of their signals, the $1-\lambda$ such investors will be pushed out of the market by the λ overconfident investors if *short-sale constraints* are introduced in the model above. This is because the necessary condition for *not* overconfident traders to stay out of the market (this being equivalent to their valuation of the asset being lower than the prevailing price), that is $\alpha_d(F+\varepsilon)<P_1$, is fulfilled for all allowed parameter values. The price in this case will be set by the overconfident traders, and it will always lie above the rational price level. On the other hand, let us assume the news at *t=1* was adverse and the signal was negative. Since the overconfident – by overestimating the precision of the signal – overreact to information, their valuation will now be below the market price and it is them now, it turn, that will be pushed out of the market by the other traders. The demand functions of the two types of investors have to be revised and will now be given by:

$$D_k = \max\left\{\frac{\gamma[\alpha_k(F+\varepsilon)-P]}{\beta_k}, 0\right\}, \tag{12}$$

where $k=c,d$. The results for prices follow immediately – they are summarized in the following proposition:

Proposition 2. *At t=1, the risky asset price can fall in one of two distinct regions:*

1. The price is set by the overconfident traders and the underconfident sit out of the market when the signal is positive. The price is then given by:

$$P_1^c = \alpha_c(F+\varepsilon) = \frac{\sigma_F^2}{\sigma_F^2 + B_c^2\sigma_\varepsilon^2}(F+\varepsilon). \tag{13}$$

2. The price is set by the underconfident traders and the overconfident sit out of the market when the signal is negative. The price is then given by

$$P_1^d = \alpha_d(F+\varepsilon) = \frac{\sigma_F^2}{\sigma_F^2 + B_d^2\sigma_\varepsilon^2}(F+\varepsilon). \tag{14}$$

The price will be equal to its rational expectations value in a special case when $B_d=1$, i.e. when there was a negative signal and perfectly rational traders push the overconfident traders out of the market.

Corollary 1. *Prices exhibit momentum in the bad news region: $Cov[(\Delta P_2,\Delta P_1)|\varepsilon<0]>0$, and reversal in the good news region: $Cov[(\Delta P_2,\Delta P_1)|\varepsilon>0]<0$, where $\Delta P_2=P_2-P_1$.*

References

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