Risk Dominance vs. Boundedly Rationality in Asymmetric Volunteer's Dilemma

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Abstract

We have developed a generalized version of an asymmetric Volunteers' Dilemma (VOD) game where cost for volunteering is different among players. But the prediction by the mixed strategy contradicts with our intuition and the experimental findings. So, we further analyzed the game with risk dominance as well as two boundedly rational models, quantal response equilibrium (QRE) and level-k model. Our analyses show that risk dominance and QRE predict the same direction of plays in this game, but level-k model and inequality aversion do not.

Keywords: Volunteers' dilemma, risk dominance, quantal response equilibrium, level-k model, inequality aversion, experiment

JEL classification number: C72, C92, D72, D74, H41

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1. Introduction

The Volunteer's Dilemma game (VOD) was first formulated by Diekmann (1985) to elucidate "social dilemmas" or "social traps" broader than those covered by the prisoner's dilemma (Kawagoe et al. 2015 fully characterize all the equilibria in this game). A typical social situation is helping behavior of people witnessing an accident or crime, as best exemplified by the murder case of Kitty Genovese examined by Darly and Latané (1968). It is said that her life could have been saved if only one of the bystanders had paid a small amount of cost (e.g., making an emergency call to the police).

An interesting issue concerning VOD is the effect of the group size on the tendency to cooperate or contribute, the so called "bystander effect." A large amount of evidence has been accumulated by political scientists and psychologists to investigate factors affecting this effect (Latané and Nida, 1981).

On the other hand, Diekmann (1993) introduced asymmetry among players about cost structure and showed theoretically that in the mixed strategy equilibrium, a player with *less* cost volunteers *less* often. But this prediction seems to be counter intuitive if we consider, for example, bystander's rescue decisions in emergencies. In that situation, for rescuing a drown boy, a skilled swimmer takes action less often than not-skilled swimmers.

Actually, Diekmann (1993) and Healy and Pate (2009) conducted the laboratory experiments and confirmed that the opposite was really the case. That is, a player with *less* cost volunteers *more* often. Diekmann (1993) suggested an explanation based on risk dominance proposed by Harsanyi and Selten (1988), but his analysis seems to be inaccurate.

So, we firstly provide rigorous analysis of the game based on risk dominance. Then, we analyze the game with two popular boundedly rational models, quantal response equilibrium (QRE) by McKelvey and Palfrey (1995) and level-k model (e.g., Crawford et al., 2013).

We showed that prediction by QRE and level-k model is completely opposite from the one of mixed strategy equilibrium under certain condition. That is, a player with *less* cost volunteers *more* often.

The organization of the paper is as follows. In the next section, we first present the model with asymmetric cost structure among players, and its mixed strategy equilibrium. Then, analyses of QRE and level-k model are shown. In Section 3, our findings are presented by using numerical examples. We conclude in the final section.

2. Model

In the volunteer's dilemma, if at least one of n players volunteers, public goods is provided. Benefit from public goods for player i is V_i, cost for volunteering for player i is K_i. When public goods is not provided, payoff for player i is L_i. Assume $V_i - L_i > K_i$ for all i.

Each player chooses volunteering (C) or not volunteering (N). Obviously, there are multiple pure strategy Nash equilibria where one and only one player chooses C and the rest of players choose N.

2.1 Mixed strategy equilibrium

Next, we consider mixed strategy equilibrium. Here, for player i, probability of choosing C is p_i and the probability of choosing N is $q_i = 1 - p_i$. Here we consider a special case in which there are only two different values for benefit V_i and cost K_i. Following Diekmann (1993), we call the players with low cost *strong* player and the players with high cost *weak* players.

As for the number of strong players, m, only m =1 case is considered in Diekmann (1993). Here we generalize those analyses to arbitrary m $(1 \leq m \leq n)$. Without loss of generality, we assume that players 1 to m are strong and the rest of players are weak.

We assume that benefit for strong player V_S is greater than or equal to the one for weak player V_W $(V_S \geq V_W)$, and that cost for strong player K_S is strictly smaller than the one for weak player K_W $(K_S < K_W)$.

Then, the mixed strategy probability of not volunteering for strong player becomes

$$q_S = \left(\frac{V_S - L_S}{K_S}\right)\left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}}.$$

Similarly, the mixed strategy probability of not volunteering for weak player is as follows.

$$q_W = \left(\frac{V_W - L_W}{K_W}\right)\left[\left(\frac{K_S}{V_S - L_S}\right)^m \left(\frac{K_W}{V_W - L_W}\right)^{n-m}\right]^{\frac{1}{n-1}}.$$

Then, we have a generalization of Diekmann (1993)'s theory for arbitrary number of strong players in a group.

Proposition 1. *The probability of volunteering of a strong player* p_S *is less than the* *one of weak player* p_W *for any m* $(1 \leq m \leq n)$.

$$p_S < p_W$$

So, the outcome in the volunteer's dilemma with asymmetric cost is inefficient in the sense that a strong player with less cost volunteers less often than weak players.

2.2 Equilibrium selection based on risk dominance

In this game, there are two kinds of pure strategy Nash equilibria, S-equilibrium where only one of the strong player chooses *C*, and W-equilibrium where only one of the weak player choses *C*. For the case of *m* = 1, Diekmann (1993) suggests that S-equilibrium is only risk dominant equilibrium. But it is not the case for some parameter values.

Without loss of generality, we assume here $L_S = L_W = 0$. If S-equilibrium risk dominates W-equilibrium, the product of deviation loss for the former is greater than that for the latter (Harsanyi and Selten, 1988). This implies

$$\frac{V_S - K_S}{K_S} > \frac{V_W - K_W}{K_W}$$

But for some parameter values, the above inequality does not hold. So, we need another rationale for our intuition that a strong player with less cost volunteers more often than weak players.

2.3 Level-k model

Level-k model is a non-equilibrium model that reflects strategic thinking by boundedly rational players. It assumes that each player adopts a strategy that corresponds to some level of strategic thinking. Level-k models have so far been applied to many games, and have succeeded in explaining a number of anomalous behaviors found in the laboratory (for survey, see Crawford et al., 2013).

Assume that $L0$ player, who is the least rational players, chooses *C* and *N* with probability 1/2 respectively. We also assume $L_S = L_W = 0$. Then, $L1$ player's best response to $L0$ is to choose C if and only if

$$V_i \geq 2^{n-1} K_i.$$

As $K_S < K_W$, if $V_S = V_W$, which is assumed in Diekmann (1993), strong player chooses *C* more likely than weak player does. So, level-k model can predict that S-equilibrium is more likely observed.

2.4 Quantal response equilibrium (QRE)

Quantal response equilibrium (QRE) is an equilibrium concept based on boundedly rational strategic behavior, assuming that players play a noisy best response (McKelvey and Palfrey, 1995). We focus on symmetric equilibria. For a parameter $\lambda \in [0, \infty)$, the stochastic best response in terms of choice probability of C is given by

$$p_i = \frac{\exp(\lambda \cdot E_i(C))}{\exp(\lambda \cdot E_i(C)) + \exp(\lambda \cdot E_i(N))} = \frac{1}{1 + \exp[\lambda \cdot \{E_i(N) - E_i(C)\}]}.$$

where $E_i(C)$ and $E_i(N)$ are the expected payoff of strategy C and N. QRE is a fixed point of this mapping.

The parameter $\lambda \in [0, \infty)$ represents the degree of rationality such that $\lambda = 0$ implies complete randomizing over pure strategies. If $\lambda = 0$, then $p = 1/2$, which is usually called the centroid of the simplex of the strategy space. McKelvey and Palfrey (1995) show that (1) the correspondence $\mathrm{QRE}(\lambda)$ is upper hemicontinuous, (2) the number of QREs is odd for generic values of λ, (3) generically, the graph $(\lambda, \mathrm{QRE}(\lambda))$ contains a unique branch which starts at the centroid and converges to a unique Nash equilibrium as λ goes to infinity. The limiting point of this principal branch is called limiting (logit) QRE. Thus, limiting QRE can serve as an equilibrium selection criterion. To obtain limiting QRE, one needs to run numerical simulation. So, we will show the results in the next section.

3. Numerical analysis

In the following numerical analysis, we fix $K_S = 20$, $K_W = 40$ and $L_S = L_W = 0$. Please note that any of the following numerical settings, S-equilibrium is always risk dominant.

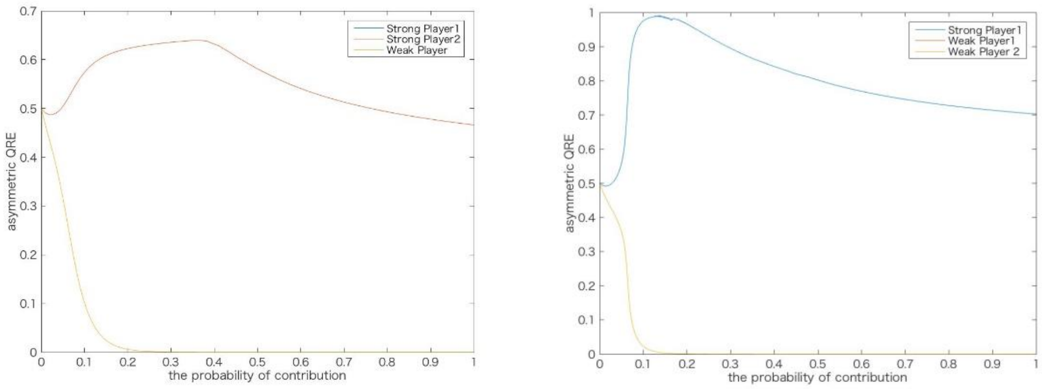
Case 1. $V_S = 60$ and $V_W = 100$.

In this case, as $V_S > 2K_S$ and $V_W > 2K_W$, both $L1$ strong and weak player's best response are C. So, level-k model predicts that both S- and W-equilibria equally likely occur. Figure 1 shows QRE correspondences for (a) two strong players and one weak player case and (b) one strong player and two weak players case. As you can see,

Observation 1. *Strong players more likely play C in both cases.*

But the choice probability of C is slightly lower in (b) than in (a). Thus,

Observation 2. *The number of strong players increases, their choice probability of C decreases.*



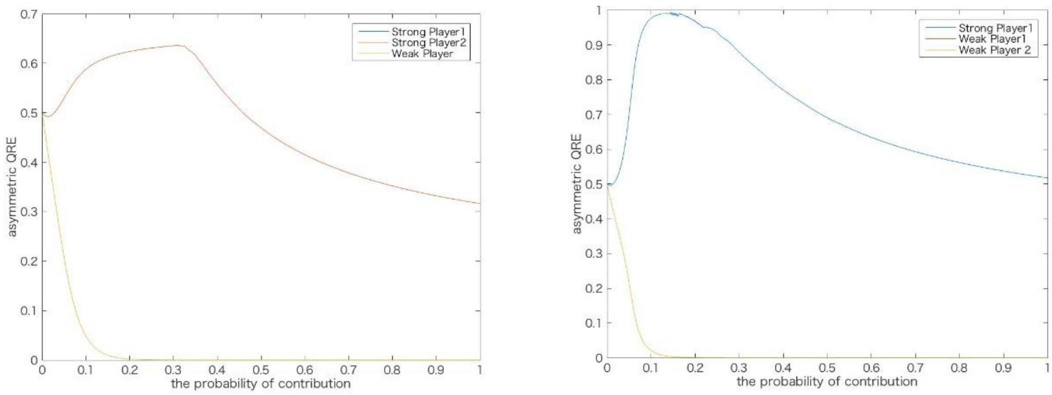
(a) **(b)**

Figure 1. QRE in case 1.

But when we focus on equilibrium payoff, as $V_W = 100 > V_S - K_S = 40$ at S-equilibrium and $V_W - K_W = 60 = V_S$ at W-equilibrium, S-player might prefer W-equilibrium because it provide equal payoff for both players (in two-person case).

Case 2. $V_S = 60$ and $V_W = 60$.

In this case, as $V_S > 2K_S$ and $V_W < 2K_W$, level-k model predicts that S-equilibrium occurs more likely. Figure 2 shows QRE correspondences for (a) two strong players and one weak player case and (b) one strong player and two weak players case. Observations 1 and 2 are both verified in this case.



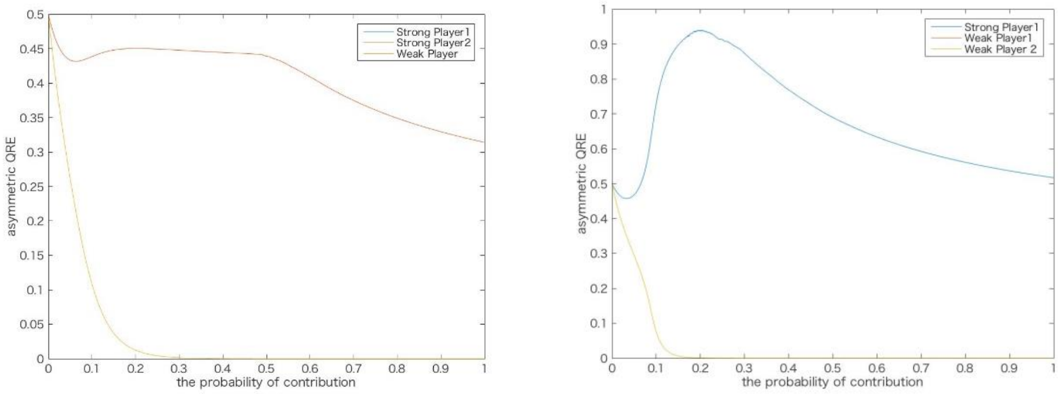
(a) **(b)**

Figure 2. QRE in case 2.

But as $V_S - K_S = 60 = V_W$ at S-equilibrium, S-players might more likely play C if they are inequality averse.

Case 3. $V_S = 35$ and $V_W = 60$.
In this case, as $V_S < 2K_S$ and $V_W < 2K_W$, level-k model predicts that neither S- nor w-equilibrium occurs. Figure 3 shows QRE correspondences for (a) two strong players and one weak player case and (b) one strong player and two weak players case. Observations 1 and 2 are both verified in this case again. Remarkable observation here is

Observation 3. *Whereas the choice probability of C for strong player is significantly lower in (a), QRE correspondence is very similar in (b) with other cases.*



(a) **(b)**

Figure 3. QRE in case 3.

Moreover, when we focus on equilibrium payoff, as $V_S - K_S = 15 < V_W = 60$ at S-equilibrium, though one may think that S-player less likely plays *C* in this case, QRE predicts the opposite, that is, S-equilibrium is likely.

5. Conclusion

We have analyzed a generalized version of an asymmetric Volunteers' Dilemma (VOD) game where cost for volunteering is different among players. But the prediction by the mixed strategy contradicts with our intuition and the experimental findings. So, we further analyzed the game with risk dominance as well as two boundedly rational models, quantal response equilibrium (QRE) and level-k models. Our analyses show that risk dominance and QRE predict the same direction of plays in this game, but level-k model and inequality aversion do not. For answering the question of which predictions may have empirical support, it is necessary to run the experiment. It will be our next research.

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