Preferences for Randomization and Anticipation*

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Abstract

In decision theory, it is not generally postulated that decision makers randomize their choices. In contrast, in real life, even though it is not realistic that decision-making is always stochastic, decision-making cannot be always deterministic. To figure out a class of preferences for randomization, we develop an axiomatic model of decision-making under a type of taste uncertainty. This model includes the model of Kreps (1979)'s subjective state spaces as a special case, and identifies the effect of randomization. In addition, we characterize a preference for the desire to randomization, and a preference for the aversion to randomization, respectively. Moreover, we generalize the model that allows for menu-dependent random choices.

KEYWORDS: Preferences for Randomization; Aversion to Randomization; Desire to Randomization; Menu-Dependence; Subjective State Spaces.

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1 Introduction

Recently, the study of stochastic choices has been developing since observed choice behaviors in various choice environments seem to be random. In fact, the issue of "indecisiveness" or "indifference" between comparable alternatives and the trade-off between "exploitation" and "exploration" may exhibit the desire to randomization in mind.[1] [2]

The goal of this paper is to, by studying random choice behaviors from the viewpoint of a type of taste uncertainty, identify a class of preferences for randomization, which is different from the Kreps (1979)'s Preferences for Flexibility or the Gul and Pesendorfer (2001)'s Self-Control Preferences. We develop an axiomatic model of decision-making under "taste uncertainty" that includes the model of the Kreps (1979)'s subjective state spaces as a special case, and identifies the effect of randomization.

The contribution of this paper is that we identify the effect of randomization. The axiomatization figures out a class of preferences for randomization with the Kreps (1979)'s framework of subjective state spaces. The representation theorem (Theorem 1) is a generalization of Kreps (1979) by relaxing Strategic Rationality and Independence.[3] In addition, we characterize a preference for the desire to randomization, and a preference for the aversion to randomization, respectively. Moreover, we generalize the model that includes menu-dependent random choices.

2 Related Literature

In decision theory, the direction of the study of stochastic choices is categorized in the following three branches: (i) preferences for flexibility, (ii) bounded rationality, and (iii) deliberate randomization. Each branch is regarded as the reason why observed choices seem to be stochastic.

First, "Preferences for Flexibility" means that decision makers prefers larger menus (choice sets) since decision makers do not perceive their own tastes at the time they choose menus.[4] [5] With preferences for flexibility, it is postulated that decision makers choose alternatives after their subjective states are realized, so the resulting behaviors are state-dependent. Thus, observed choices seem stochastic since subjective states themselves are not observable.[6]

Second, "Bounded Rationality" is also regarded as the reason for random choice behaviors.

[1] The issue of indecisiveness and indifference is studied by Eliaz and Ok (2006). Eliaz and Ok (2006) analyze the issue by choice correspondences as primitives.

[2] The trade-off between exploitation and exploration is classically studied in statistical decision theory. Bergemann and Valimaki (2006) provides a survey for economic applications. In choice settings, "exploitation" is interpreted as choosing the same alternatives repeatedly in a decision problem. On the other hand, "exploration" means that a decision maker tries to make a different decision in a decision problem.

[3] Strategic Rationality is stated as follows. Let $\mathcal{A}$ be the set of menus. For any $A, B \in \mathcal{A}$, if $A \succsim B$, then $A \sim A \cup B$.

[4] Let $\mathcal{A}$ be the set of all menus. Formally, preferences for flexibility is stated as follows: for any menus $A, B \in \mathcal{A}$, $B \subseteq A$ implies $A \succsim B$.

[5] This framework stems from Kreps (1979), and is called *subjective* state spaces. Subjective states captures the decision maker's taste uncertainty. Each state is seen as a mood or feeling.

[6] See Kreps (1979), Dekel et al. (2001), and Ahn and Sarver (2013). Dean and McNeill (2014) provides an experimental evidence of the relationship between preferences for flexibility and random choices.

Even though decision makers have complete and transitive preferences, decision makers may not always chooser their best alternatives because of the limitation of cognitive abilities.[7]

Third, the pattern of stochastic choices is due to the result of “Deliberate Reasoning.” This idea implies that decision makers have a class of preferences for randomization. Recent experimental evidence studied by Agranov and Ortoleva (2015) and Dwenger et al. (2014) indicates that preferences for the desire of randomization are supported. [8] Cerreia-Vioglio et al. (2015) presents a random choice representation due to deliberate reasoning by stochastic choice functions as primitives.

This paper is categorized in the third approach. By using the framework of subjective state spaces, we identify a class of preferences for randomization. We also identify not only preferences for the desire to randomization, but also preferences for the aversion to randomization. Moreover, since the attitude toward randomization in mind is changeable based on menus, we generalize the model that includes menu-dependent random choices.

3 Model

Let X be a set of all alternatives. Assume that X is the compact and convex subset in $\mathbb{R}^N$ where N is a positive integer, endowed with the Euclidean metric d.[9] Alternatives are denoted by $x, y, z \in X$. Let $\mathcal{A}$ denote the set of all non-empty compact subsets of X, endowed with the Hausdorff metric. The elements in $\mathcal{A}$ are called *menus*. Menus are denoted by $A, B, C \in \mathcal{A}$.

The primitive of the model is a binary relation $\succsim$ over $\mathcal{A}$. The asymmetric and symmetric parts of $\succsim$ are denoted by $\succ$ and $\sim$ respectively.

3.1 Axioms

First, we provide a standard requirement in decision theory.

Axiom (Standard Preferences): $\succsim$ is (i) a *weak order* and (ii) *continuous*, and (iii) *non-degenerate*:

(i) (Weak Order): $\succsim$ is *complete* and *transitive*.

(ii) (Continuity): The sets $\{A \in \mathcal{A} \mid A \succsim B\}$ and $\{A \in \mathcal{A} \mid B \succsim A\}$ are closed.

(iii) (Non-Degeneracy): There exists $A, B \in \mathcal{A}$ such that $A \succ B$.

Next, let us consider Strategic Rationality: $A \succsim B \Rightarrow A \sim A \cup B$. Preferences for randomization might violate Strategic Rationality since the randomization on the menu $A \cup B$ can produce the incentive of choosing alternatives from the menu B. To avoid this, we relax Strategic Rationality. Strategic Rationality holds only if there exists an alternative in a menu A that dominates all alternatives in a menu B.

[7]See, for example, Manzini and Mariotti (2014), which studies random choices in terms of consideration sets.

[8]Agranov and Ortoleva (2015) states that, in the same question multiple times, subjects try to report stochastic answers. Dwenger et al. (2014) suggest that decision makers randomize between alternatives as a desire to minimize regret.

[9]This assumption is due to study the effect of *randomization*. N is interpreted as the number of attributes of alternatives.

Axiom(Monotonic Strategic Rationality): If for any $y \in B$, there exists $x \in A$ such that $x \geq y$, then $A \succsim B$.

Moreover, we introduce the following axiom, which is related to preferences for hedging. The axiom says that if hedging alternatives is beneficial, then the decision maker prefers randomizing them subjectively.

Axiom(Hedging): For any $x, y, z \in X$ and $\lambda \in [0, 1]$, if $\{\lambda x + (1 - \lambda)y\} \succsim \{z\}$, then $\{x, y\} \succsim \{z\}$.

Finally, we provide a weaker version of Independence. We say that a menu A dominates another menu B if for any $y \in B$ there exists $x \in A$ such that $x \geq y$.

Axiom(Domination Independence): For any $A, B, C \in \mathcal{A}$ such that A dominates B and any $\lambda \in [0, 1]$, $\lambda A + (1 - \lambda)C \succsim \lambda B + (1 - \lambda)C$.

3.2 Representation

We present the anticipated utility representation of optimal random choices.

Theorem 1. *The following statements are equivalent:*
(a) $\succsim$ satisfies Standard Preferences, Monotonic Strategic Rationality, Hedging, and Domination Independence.
(b) There exists a continuously strictly increasing function $u : X \to \mathbb{R}$ such that $\succsim$ is represented by $U : \mathcal{A} \to \mathbb{R}$ defined by

$$U(A) = \max_{\rho \in \Delta(A)} u\Big(\sum_{x \in A} x\rho(x)\Big),$$

where $\Delta(A)$ is the set of finite Borel probability measures on A such that $\sum_{x \in A} \rho(x) = 1$, $\rho(x) \in [0, 1]$ for any $x \in A$.

We call $\succsim$ a random anticipated utility representation if $\succsim$ satisfies the axiom in Theorem 1. We present the uniqueness result as follows.

Proposition 1. *u is unique up to positive affine transformations.*

3.3 Characterization of u

We characterize the utility function u. First, we characterize preferences for the desire to randomization and preferences for the aversion to randomization, respectively.

Axiom(Desire to Randomization): If $\{x\} \sim \{y\}$, then for any $\lambda \in [0, 1]$, $\{\lambda x + (1 - \lambda)y\} \succsim \{x\}$.

Axiom(Aversion to Randomization): If $\{x\} \sim \{y\}$, then for any $\lambda \in [0, 1]$, $\{x\} \succsim \{\lambda x + (1 - \lambda)y\}$.

Proposition 2. *Suppose that a random anticipated utility representation is represented by u. Then, the following statements hold:*

(i) u is concave if and only if $\succsim$ exhibits Desire to Randomization;

(ii) u is convex if and only if $\succsim$ exhibits Aversion to Randomization.

Next, we consider the Independence axiom:

Axiom(Independence): For any $A, B, C \in \mathcal{A}$ and any $\lambda \in [0,1]$, $A \succsim B \Leftrightarrow \lambda A + (1-\lambda)C \succsim \lambda B + (1-\lambda)C$.

We say that the utility function u is linear if for any $x, y \in X$ and $\lambda \in [0,1]$, $u(\lambda x + (1-\lambda)y) = \lambda u(x) + (1-\lambda)u(y)$.

Proposition 3. *Suppose that a random anticipated utility representation is represented by u. Then, u is linear if and only if $\succsim$ satisfies Independence.*

4 Menu-Dependent Random Choices

In general, the attitude toward randomization in mind is changeable based on menus. We generalize the random anticipated utility representation that allows for menu-dependence.

Axiom(Randomization I): If $x \geq y$, then $\lambda\{x,y\} + (1-\lambda)\{z\} \sim \lambda\{x\} + (1-\lambda)\{z\}$ for any $\lambda \in [0,1]$ and $z \in X$.

Axiom(Randomization II): If $\{x,y\} \sim \{z\}$, then the following holds:

(i) $z' \geq z \Rightarrow \lambda\{x,y\} + (1-\lambda)\{z'\} \succsim \lambda\{z\} + (1-\lambda)\{z'\}$;

(ii) $z' \leq z \Rightarrow \lambda\{x,y\} + (1-\lambda)\{z'\} \precsim \lambda\{z\} + (1-\lambda)\{z'\}$.

We have the menu-dependent random anticipated utility representation. We write down the function $u : X \times \mathcal{A} \to \mathbb{R}$ by $u_A(x) := u(x, A)$ for any $x \in X$ and $A \in \mathcal{A}$.

Proposition 4. *The following statements are equivalent:*
(a) $\succsim$ satisfies Standard Preferences, Monotonic Strategic Rationality, Hedging, Domination Independence, Randomization I, and Randomization II.
(b) There exists a continuously strictly increasing function $u : X \times \mathcal{A} \to \mathbb{R}$ such that $\succsim$ is represented by $U : \mathcal{A} \to \mathbb{R}$ defined by

$$U(A) = \max_{\rho \in \Delta(A)} u_A\Big(\sum_{x \in A} x\rho(x)\Big),$$

where $\Delta(A)$ is the set of finite Borel probability measures on A such that $\sum_{x \in A} \rho(x) = 1$, $\rho(x) \in [0,1]$ for any $x \in A$.

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