A Continuous-Time Agency Model under Loss Aversion*

Keiichi Hori[†] **and Hiroshi Osano**[‡]

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Abstract

A continuous-time agency model is explored where the agent has loss-aversion preferences. We show that the optimal contract includes a flat part insensitive to the agent's continuation payoff, whereas the flat part is preceded and followed by a range of option-type payoffs. Furthermore, the introduction of loss aversion induces the investors to reward the agent earlier, and to use a higher-powered incentive scheme. Implementing the optimal contract by standard securities, we provide possible explanations for the evolution of CEO compensation with the low level of stability of CEOs' equity ownership in the United States, and for the corporate dividend-smoothing policy.

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Correspondence to: Hiroshi Osano, Institute of Economic Research, Kyoto University, Sakyo-ku, Kyoto 606-8501, Japan.

E-mail: osano@kier.kyoto-u.ac.jp; **Tel:** +81-75-753-7131; **Fax:** +81-75-753-7138.

[†]Faculty of Economics, Ritsumeikan University

[‡]Institute of Economic Research, Kyoto University

1. Introduction

The evolution of CEO compensation in the United States since World War II can be divided broadly into three distinct periods (see Frydman and Jenter (2010) and Frydman and Saks (2010)). Prior to the 1970s, CEO compensation was characterized by low levels of pay, little dispersion across top managers, and moderate pay–performance sensitivities. From the mid-1970s to the end of the 1990s, CEO compensation changed considerably: compensation levels trended upward dramatically, differences in compensation across managers and firms increased, and, in particular, stock options grew substantially to become the single largest component of CEO compensation in the 1990s. The sensitivity of CEO wealth to firm performance also surged in the 1990s, mainly because of rapidly growing option portfolios. During the 2000s, CEO compensation shifted again: average CEO compensation declined, and restricted stock grants replaced stock options as the most popular form of stock compensation. By contrast, most CEOs' fractional equity ownership remained low throughout these three periods, although it has increased gradually. Dividend smoothing also is one of the most widely documented phenomena in the corporate financing literature, because firms' primary concern is the stability of dividends (see Leary and Michaely (2011)).

To provide possible explanations for these stylized facts, we develop a continuous-time agency model in which the agent has loss-aversion preferences, as introduced by prospect theory (Kahneman and Tversky (1979)). Our model represents a significant departure from the previous continuous-time agency literature in that the agent derives gain–loss utility from comparing his actual consumption with a reference point, instead of his absolute level of consumption. We first address the following theoretical questions. (i) Is there a range of the agent's continuation payoff—the total payoff that investors (the principal) expect(s) the agent to derive in the future after a given moment in time—in which he is rewarded with fixed cash compensation? (ii) Is there a range of the agent's continuation payoff in which his fixed cash compensation is reduced according to performance or where all excess cash flows are paid immediately as the exercise of option grants? In addition, is there any difference in the sensitivity of the agent's compensation to performance in these nonflat ranges? (iii) Does the introduction of loss aversion induce investors to reward the agent earlier? (iv) How do the key parameters such as the degree of loss aversion or the magnitude of the agency problem affect the incentive scheme? In particular, does an increase in the degree of the agent's loss aversion induce investors to use a lower- or higher-powered incentive scheme? (v) How can the resulting optimal contract be implemented with a capital structure (a line of credit, long-term debt, and equity) in which the agent controls the payout policy. Exploiting the results of these theoretical questions, we can provide explanations for the stylized facts mentioned at the beginning: (i) the evolution of CEO compensation in the United States since World War II—in particular, the rise in CEO compensation and the use of equity-linked pay since the 1970s—as well as the low level of stability of CEOs' fractional equity ownership in the same period; and (ii) the corporate dividend-smoothing policy.

Our model generalizes the continuous-time agency model of DeMarzo and Sannikov (2006) by incorporating loss aversion. We consider a continuous-time setting in which a risk-neutral agent with limited liability and loss aversion needs to raise external capital from risk-neutral investors in order to start up a project and cover future operating losses. However, the agent can divert the cash flows from the project for personal consumption by taking hidden actions.[1] The agent also derives gain–loss utility from comparing his actual consumption level with his reference consumption level. The agent and investors then sign a contract, which obliges the agent to report the cash flows from the project to investors and specifies transfers between the agent and investors and the date at which investors can terminate the project. To provide the agent with appropriate incentives, investors control the transfers to the

[1] As shown in DeMarzo and Sannikov (2006), the characterization of the optimal contract in our model is unchanged even though the agent makes a hidden effort choice as in the standard principal–agent model.

agent, and may also force the early termination of the project, conditional on the agent's continuation payoff. Under the optimal contract, the agent finds it optimal to report the cash flows truthfully. The contract is terminated when the agent's continuation payoff equals his reservation payoff.

In the original model of DeMarzo and Sannikov (2006), the risk-neutral agent is more impatient than the investors. Because exchanging relative consumption timings between the agent and investors improves efficiency, the optimal contract induces investors to pay cash to the agent as early as possible. However, paying cash to the agent earlier reduces his continuation payoff, and will make future inefficient liquidation more likely. Thus, the optimal contract requires investors to set an optimal cash payment threshold (reflecting barrier) of the agent's continuation payoff below which the agent is not paid, but above which all excessive cash flows are immediately paid to the agent. The observed compensation contract, however, is at odds with these predictions because it is a combination of both a fixed part (high base salaries) where payment does not vary with performance and a flexible part (option holdings) where payment varies with performance once a certain level of performance is achieved.

To provide possible explanations for the observed practice, we incorporate loss aversion into the agent's preferences. The key feature is that the agent's utility depends partly upon the deviation of consumption from a reference level: the agent compares his actual consumption level with the reference consumption level. Then, there is a kink in the utility function at the reference consumption with the utility function steeper immediately below this level. If the agent attains only a low level of consumption, he compares this with the higher consumption he could have attained, and experiences the sensation of a loss from this comparison. The anticipation of these losses reduces the agent's continuation payoff. Thus, loss aversion—the agent's higher sensitivity to losses than to gains around the reference point—induces the agent to be more risk averse near the reference point; this may therefore generate partial incentives for the agent to avoid staying in the loss space where his actual consumption level is not larger than his reference consumption level. However, the kink in the utility function causes our dynamic continuous-time optimization problem to be extremely difficult. Therefore, we need to overcome the problem of the kinked utility function in the dynamic system in order to discuss whether loss aversion creates a fixed segment where positive payment does not vary with performance, and a flexible segment where payment varies with performance once a certain level of performance is achieved.

The main results are as follows. Our first main result is that the optimal contract includes a range of the agent's continuation payoff where he is rewarded with fixed cash compensation. At each end of the flat range, there is a range consisting of the lower levels of the agent's continuation payoff where the fixed cash compensation is reduced according to his continuation payoff, and a range consisting of the higher levels of his continuation payoff where all excess cash flows are paid immediately as the exercise of option grants (for example, the exercise of stock option awards with performance-based vesting provisions). The sensitivity of the agent's compensation to his continuation payoff is lower in the former nonflat range than in the latter one. Furthermore, both the beginning of the former nonflat range and the beginning of the latter nonflat range minus the reference consumption level are absorbing states along the optimal path.

Intuitively, note that the agent's impatience implies a strictly positive marginal benefit of investors paying cash to the agent earlier. However, as the earlier payment to the agent may cause future inefficient liquidation, the optimal contract requires investors to set a cash payment threshold at which investors start paying the cash to the agent. Now, the incorporation of loss aversion results in the agent's higher sensitivity to losses than to gains. Because entering the loss space is an expensive way to compensate the agent, investors have an incentive to increase payments to the agent in the loss space in order to reduce the scope for incurring a loss or to reduce disutility from loss aversion in the loss space. Thus, the agent's cash compensation varies with his continuation payoff once his continuation payoff exceeds the cash payment threshold in the loss space. In fact, loss aversion does not operate in the gain space. Hence, if the agent's compensation exceeds the reference level of income when his continuation payoff increases, investors' incentives to pay any additional cash are mitigated. As a result, if the agent's continuation payoff is large enough, investors make payments exactly equal to the reference level of income so that they reduce disutility from loss aversion as much as possible but avoid leaving the agent in the gain space. However, if the agent's continuation payoff is even larger, he enters a cash payment threshold in the gain space, where all excess cash flows are paid immediately as the exercise of option grants. Because the agent stays in the gain space and feels no loss aversion, the benefit to the agent of receiving cash is greater. Hence, the sensitivity of the agent's compensation to his continuation payoff is higher in this range than in the other ranges.

The second main result is that the introduction of loss aversion induces investors to reward the agent earlier. Intuitively, the agent's impatience implies a strictly positive marginal benefit of investors paying cash to the agent earlier, while the possibility of future inefficient liquidation creates a marginal cost of investors paying cash to the agent earlier. In the absence of loss aversion, the optimal contract requires investors to set the timing of the cash payments so that the marginal benefit equals the marginal cost. However, the introduction of loss aversion creates the additional benefit of investors paying cash to the agent earlier because a positive payment to the agent reduces disutility from loss aversion in the loss space. Hence, this additional effect induces investors to pay cash earlier in the presence of loss aversion than in its absence.

Our third main result shows that an increase in the agent's degree of loss aversion induces investors to use a higher-powered incentive scheme. Intuitively, underreporting income and diverting the cash flows from the project reduce disutility from loss aversion in the loss space at a rate that is proportional to the degree of loss aversion. Thus, with a higher degree of loss aversion, investors need to provide a greater incentive to the agent. By contrast, the static moral hazard model of Herweg, Müller, and Weinschenk (2010) indicates that an increase in the agent's degree of loss aversion may allow the principal to use weaker incentives. The difference is that in their model, a higher degree of loss aversion may be associated with a stronger incentive for the agent to choose a high effort in order to reduce the scope for incurring a loss by affecting the probability distribution of the outcome. This possibility allows the principal to use weaker incentives. However, our result can also be extended to the hidden action model if the hidden action directly affects the firm's cash flow level rather than the probability distribution of the firm's cash flow.

The fourth main result relates to capital structure implementation. As in DeMarzo and Sannikov (2006), the optimal contract is implemented by a combination of equity, long-term debt, and a line of credit. The credit line balance traces the agent's continuation payoff. The agent is compensated by holding a fraction of the firm's equity and receiving dividends, whereas the remaining equity, debt, and line of credit are held by outside investors. However, the capital structure implementation of our model is different from that of DeMarzo and Sannikov (2006) in the following respects: (i) the percentage of the firm's equity held by the agent is increasing in the agent's degree of loss aversion; (ii) dividends are paid before the line of credit has been paid off; (iii) dividend payments are positive but insensitive to the firm's performance if the credit line balance is sufficiently low; (iv) a large line of credit delays or reduces dividend payments; and (v) the total debt capacity of the firm is sensitive to the volatility of the project returns and the liquidation value of the project.

As mentioned at the beginning, our theoretical findings yield empirical implications for the evolution of CEO compensation as well as the low level of stability of CEOs' fractional equity ownership in the United States since World War II, and for the corporate dividend-smoothing policy. First, many theories have been developed to explain the rise in CEO compensation and the use of stock-based compensation since the 1970s. For example, the managerial power (rent extraction) theory suggests that the high level of CEO compensation is the result of executives' ability to set their own pay and extract rents from the firms they manage (see Bebchuk and Fried (2003, 2004)). The competitive pay theory also indicates that the high level of CEO compensation is viewed as the efficient outcome of a managerial labor market where firms compete optimally for managerial effort or talent (see Frydman and Jenter (2010) for a review of the literature). However, none of these theories provides a compelling explanation for the apparent change in CEO compensation that occurred during the 1970s, or for the rise in CEO compensation and the use of equity-linked pay since the 1970s together with the low level of stability of CEOs' fractional equity ownership in the same periods. Furthermore, neither theory explains the explosive growth of options in the 1990s and their recent decline in favor of restricted stock. By contrast, our model sheds new light on these problems from a dynamic perspective using the loss-aversion framework. In particular, along the evolution of the agent's continuation payoff, our model derives not only the optimality of fixed cash compensation but also the optimality of option-like compensation, together with the low level stability of the agent's fractional equity ownership. As the agent's continuation payoff can be viewed as equity value to outside investors, the evolution of equity value can provide some explanations for the stylized facts of the evolution of CEO compensation and the low level of stability of CEOs' fractional equity ownership in the United States since World War II. Second, although several recent theoretical studies have suggested a renewed interest in explaining dividend smoothing, there is little agreement as to why firms smooth their dividends or what determines a firm's propensity to smooth (see Leary and Michaely (2011) for a survey of the literature). Our theoretical results give some explanation of dividend smoothing on the basis of loss aversion under the continuous-time agency framework.

1.1. Related literature.—

The work in this paper is related to the growing literature on continuous-time principal–agent models using martingale techniques. DeMarzo and Sannikov (2006), Philippon and Sannikov (2007), Hoffmann and Pfeil (2010), and Piskorski and Tchistyi (2010) analyze a cash-diversion model with a risk-neutral agent, whereas Biais, Mariotti, Rochet, and Villeneuve (2010) study a large risk-prevention model with a risk-neutral agent. He (2009) extends the continuous-time agency model by allowing a risk-neutral manager to control privately the drift of the geometric Brownian motion firm size. Sannikov (2008), Jovanovic and Prat (2010), and He (2011) examine an agency problem with a risk-averse agent in a firm whose cash flows are determined by the agent's unobservable effort.

The main difference between our model and the aforementioned continuous-time agency models is that our model deals with an agent with loss-aversion preferences. As a result, even under the cash-diversion model à la DeMarzo and Sannikov (2006), we show that the optimal contract includes a range of the agent's continuation payoff in which he is rewarded with fixed cash compensation. In addition, our model provides a theoretical underpinning for the use of option-like incentive schemes in CEO compensation. These features enable us to give explanations not only for the evolution of CEO compensation in the United States since World War II as well as the low level of stability of CEOs' fractional equity ownership in the same period, but also for the corporate dividend-smoothing policy.

Our work is also related to the static principal–agent literature that incorporates loss aversion as introduced by the prospect theory of Kahneman and Tversky (1979). On the basis of experimental evidence, Kahneman and Tversky propose a value function defined according to the gains or losses relative to a reference point, instead of the absolute level of consumption or wealth.[2] Based on several notions of loss aversion, de Meza and Webb (2007) show that there will be intervals over which pay is insensitive to performance; they also rationalize the use of option-like compensation. Incorporating the expectation-based loss aversion of Kőszegi and Rabin (2007), Herweg, Müller, and Weinschenk (2010) indicate that the optimal contract is a binary pay-

[2] The value function is defined according to deviations from the reference point, is generally concave for gains and convex for losses, and is steeper for losses than for gains.

ment scheme (lump-sum bonus contract). Calibrating a stylized principal–agent model, Dittmann, Maug, and Spalt (2010) suggest that the loss-aversion model generates convex compensation contracts and dominates an equivalent risk-aversion model in explaining the observed compensation contracts of 595 CEOs in the United States.

The commonality of the loss aversion concepts in these static models is that there is typically a range in which a positive payment does not vary with performance. Even though our model deals with a dynamic, continuous-time agency setting, we also succeed in deriving an optimal contract that includes a range of the agent's continuation payoff in which he is rewarded with fixed cash compensation. De Meza and Webb (2007) and Dittmann, Maug, and Spalt (2010) derive the optimality of option-type contracts by assuming the agent's risk aversion or risk tolerance. However, we show that even if the agent is risk neutral, option-type contracts are optimal under the interaction of the agent's loss aversion with the dynamic factors related to the agent's impatience and the possibility of inefficient future liquidation. Furthermore, the static principal–agent model with loss aversion does not explain the low level of stability of CEOs' fractional equity ownership nor the corporate dividend-smoothing policy.

In the dynamic context, Kyle, Ou-Yang, and Xiong (2006) solve a liquidation problem when a project owner has loss-aversion preferences. However, as they use the real options approach in a single agent's maximization model, their model cannot deal with the principal–agent problem; as a result, their model cannot consider how the optimal compensation contract should be designed.

The paper is organized as follows. Section 2 describes the basic model. Section 3 derives an optimal contract. Section 4 discusses the implementation of the optimal contract. Section 5 outlines the empirical implications of our results. The final section contains the conclusions. The proofs for all the lemmas and propositions are summarized in the Appendix.

2. The Model

We present a continuous-time principal–agent model in which risk-neutral investors of an infinitely lived firm hire a risk-neutral agent to operate the firm. The firm produces the following cash flows:

$$dY_t = \mu dt + \sigma dZ_t, \tag{1}$$

where μ is the drift of the cash flows, σ is the volatility of the cash flows, and $Z = \{Z_t, \mathcal{F}_t;\ 0 \leq t < \infty\}$ is a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, Q)$.

We assume that μ and σ are observed publicly. However, we assume that investors do not observe the cash flows $Y \equiv \{Y_t;\ 0 \leq t < \infty\}$, whereas the agent does. Thus, the agent has the opportunity to misrepresent his income. The agent reports cash flows $\widehat{Y} \equiv \{\widehat{Y}_t;\ 0 \leq t < \infty\}$ to investors, but can underreport the cash flows by diverting them or overreport the cash flows by reinvesting his own money back into the project. Based on the agent's reports, a contract, (τ, I), specifies a termination time of the relationship, τ, and compensation for the agent, $I \equiv \{I_t;\ 0 \leq t < \infty\}$.[3]

If the agent receives a fraction $\lambda \in (0,\ 1]$ of the cash flows he diverts, his total flow of income at time t equals

$$\left[dY_t - d\widehat{Y}_t\right]^{\lambda} + dI_t, \qquad \text{where } \left[dY_t - d\widehat{Y}_t\right]^{\lambda} \equiv \lambda\left(dY_t - d\widehat{Y}_t\right)^{+} - \left(dY_t - d\widehat{Y}_t\right)^{-},$$

where $\left(dY_t - d\widehat{Y}_t\right)^{+}$ represents diversion and $\left(dY_t - d\widehat{Y}_t\right)^{-}$ indicates overreporting.

The agent is risk neutral, but a negative wage is ruled out by limited liability. The agent also discounts his consumption at rate γ, and keeps a private savings

[3] τ is a $\widehat{Y}$-measurable stopping time, and I is a $\widehat{Y}$-measurable continuous process.

account. The agent's balance S_t evolves at interest rate $\rho < \gamma$ according to

$$dS_t = \rho S_t dt + \left[dY_t - d\widehat{Y}_t\right]^{\lambda} + dI_t - dC_t, \tag{2}$$

where $dC_t \geq 0$ is the agent's consumption at time t. Because the agent must maintain a nonnegative balance in his account, S_t is restricted such that $S_t \geq 0$.

The agent is assumed to have loss-aversion preferences à la de Meza and Webb (2007), Herweg, Müller, and Weinschenk (2010), and Dittmann, Maug, and Spalt (2010). Hence, the agent's total expected payoff from the contract at date 0 is represented by

$$W_0 = E\left\{\int_0^{\tau} e^{-\gamma s}\left[dC_s + \theta(dC_s, a)(dC_s - ads)\right] + e^{-\gamma\tau}R\right\}, \tag{3}$$

where $a > 0$ is the reference level and

$$\theta(dC_s, a) = \begin{cases} \theta, & \text{if } dC_s \leq ads, \\ 0, & \text{if } dC_s > ads. \end{cases} \tag{4}$$

Note that the agent receives $R \geq 0$ from an outside option when the contract is terminated.

The agent's total expected payoff has three components: consumption utility, $\int_0^{\tau} e^{-\gamma s} dC_s$; loss-aversion disutility, $\int_0^{\tau} e^{-\gamma s}\theta(dC_s, a)(dC_s - ads)$; and a termination payoff, $e^{-\gamma\tau}R$. The loss-aversion component captures the feature that the psychological pain of falling below the reference level is greater than any pleasure from surpassing it by an equal amount (see prospect theory introduced by Kahneman and Tversky (1979)). Therefore, loss aversion applies and $\theta(dC_s, a) = \theta$ in the loss space ($dC_s \leq ads$), whereas it does not operate in the gain space ($dC_s > ads$). Formally, this introduces a kink in the agent's value function at a. For the termination payoff, R may include the deduction amount in the loss space if the outside option cannot attain the reference-level consumption.

Investors can commit to an employment contract (τ, I). In response to (τ, I), the agent chooses a feasible strategy $(C, \widehat{Y})$ to maximize his total expected payoff, where $C \equiv \{C_t;\ 0 \le t < \infty\}$.[4] The agent's strategy $(C, \widehat{Y})$ is incentive compatible if it maximizes his total expected payoff (3) given (τ, I). Conversely, a contract (τ, I) is incentive compatible if it induces the agent's incentive-compatible strategy.

We need not explicitly consider the agent's option to quit or to receive the outside option R. The reason is that the agent can always underreport his income until termination at a rate that yields the agent at least R.

Investors are assumed to be risk and loss neutral, and to discount their cash flows at the market interest rate r that satisfies $\rho \le r < \gamma$. Once the contract is terminated, investors receive liquidation payoff $L \ge 0$. We assume that $L < \frac{\mu}{r}$ so that liquidation is inefficient. Because investors contribute external capital K, which is required for the project to be started, the total expected payoff of investors at date 0 is then

$$E\left[\int_0^{\tau} e^{-rs}(d\widehat{Y}_s - dI_s) + e^{-r\tau}L\right] - K. \tag{5}$$

Now, the optimal contracting problem is to find an incentive-compatible contract (τ, I) that maximizes the total expected payoff of investors subject to delivering the agent an initial required payoff W_0.

3. Optimal Contracting

In this section, we consider recursively the dynamic moral hazard problem and derive the optimal contract, employing the continuous-time techniques developed by DeMarzo and Sannikov (2006). We first show the following lemma, which ensures that it is sufficient to find an optimal contract within a smaller class of contracts.

[4] A feasible strategy is a pair of processes $(C, \widehat{Y})$ adapted to Y so that (i) $\widehat{Y}$ is continuous and if $\lambda < 1$, $Y_t - \widehat{Y}_t$ has bounded variation, (ii) C_t is nondecreasing ($dC_t \ge 0$, that is, nonnegative consumption), and (iii) the saving process of (2) stays nonnegative.

Lemma 1: *There exists an optimal contract where the agent chooses to report cash flows truthfully, and maintains zero savings.*

Thus, without loss of generality, we focus on an optimal contract in which truth telling and zero savings are incentive compatible. The intuition is similar to that given by DeMarzo and Sannikov (2006).

3.1. Optimal Contract without Savings.—

3.1.1. The agent's continuation payoff and incentive compatibility

If the agent could not save secretly, he would not be able to overreport cash flows and he would consume all his income, as shown in DeMarzo and Sannikov (2006). In this case, (2) would be rearranged so that

$$dC_t = \lambda \left[dY_t - d\widehat{Y}_t \right] + dI_t, \tag{6}$$

where $dY_t - d\widehat{Y}_t \geq 0$. However, we assume that investors do not observe the agent's consumption. Hence, $dY_t - d\widehat{Y}_t$ is not observable to investors. In Section 3.1, assuming that the agent can neither save secretly nor steal at an unbounded rate,[5] we find an optimal contract in which truth telling and zero savings are incentive compatible. In Section 3.2, we show that the optimal contract derived in Section 3.1 remains incentive compatible even though we allow the agent to save secretly or to steal at an unbounded rate.

It follows from (3) and (6) that for any contract (τ, I), we define the agent's continuation payoff at time t after a history of reports $(\widehat{Y}_s,\ 0 \leq s \leq t)$ if he tells the truth $(dY_t = d\widehat{Y}_t)$ after time t:

$$W_t(\widehat{Y}) = E_t \left\{ \int_t^\tau e^{-\gamma(s-t)} \left[dI_s + \theta(dI_s, a)(dI_s - a ds) \right] + e^{-\gamma(\tau - t)} R \right\}. \tag{7}$$

$W_t(\widehat{Y})$ is the agent's continuation value obtained under (τ, I) if he tells the truth after time t and works from t onward until the time the contract is

[5] Formally, $Y_t - \widehat{Y}_t$ is Lipschitz continuous.

terminated.

The following lemma gives the evolution of W_t in terms of reports $\widehat{Y}_t$.

Lemma 2: *For any contract (τ, I), there is a sensitivity process of the agent's continuation payoff toward his report, $\{\beta_t : 0 \leq t \leq \tau\}$, such that for every $t \in [0,\ \tau]$, his continuation value W_t evolves according to*

$$dW_t = \gamma W_t dt - dI_t + \beta_t(d\widehat{Y}_t - \mu dt) - \theta(dC_t, a)(dC_t - adt), \qquad t \in [0,\ \tau]. \tag{8}$$

The truth-telling contract is incentive compatible if and only if

$$\beta_t \geq \lambda[1 + \theta(dC_t, a)], \qquad \forall dC_t \geq 0 \text{ and } t \in [0,\ \tau]. \tag{9}$$

Equation (8) describes how the agent's continuation value must evolve over time. The first three terms on the right-hand side of (8) are the same as in DeMarzo and Sannikov (2006). The first and second terms represent a drift component that corresponds to promise keeping. These components imply that W_t has to grow at the agent's discount rate γ, less his compensation dI_t. The third term is a diffusion component that reflects the agent's reward from reporting his income. The diffusion component is related to the agent's incentives for reporting truthfully. The final term on the right-hand side of (8) is novel and captures the effect of loss aversion. It shows that the agent's continuation payoff must be compensated by $\theta(adt - dC_t)$ when loss aversion applies in the loss space ($\theta(dC_s, a) = \theta$ if $dC_s \leq ads$); by contrast, the agent's continuation payoff need not be adjusted when loss aversion does not operate in the gain space ($\theta(dC_s, a) = 0$ if $dC_s > ads$).

To make it incentive compatible for the agent to report income truthfully, the contract must make the benefits from underreporting income less than the benefits from reporting all income. For any additional unit of income, if the agent chooses to underreport income, it follows from (6) that he gains a private benefit from diversion, λ. He also reduces disutility from loss aversion, $\lambda\theta(dC_t, a)$, by consuming the diverted income. By contrast, if the agent reports all income, he gains β_t because his continuation utility is then increased by β_t. These discussions imply that the agent will report income truthfully as long as $\beta_t \geq \lambda[1 + \theta(dC_t, a)]$ for any $dC_t \geq 0$.

3.1.2. Investors' value function Let $b(W)$ be the investors' value function, which is the highest expected payoff to investors obtained from an incentive-compatible contract that provides the agent with a payoff equal to W. To simplify our discussion, we assume that $b(W)$ is concave. In Proposition 2, we verify that $b(W)$ is concave.

We first discuss a cash payment threshold where investors start paying cash to the agent. In the optimal contract, $dC_s = dI_s$. It follows from (4) and (8) that the lump-sum transfer of dI decreases W by $dI(1 + \theta)$ if $dI \leq a$, while it reduces W by dI if $dI > a$. Because investors can always pay a lump-sum transfer of dI units of income and move to the contract with $W - dI(1 + \theta)$ if $dI \leq a$ (or $W - dI$ if $dI > a$), the optimality then implies that

$$b(W) \geq \begin{cases} b(W - dI(1+\theta)) - dI, & \text{if } dI \leq a, \\ b(W - dI) - dI, & \text{if } dI > a. \end{cases}$$

The above inequality is rewritten so that

$$b'(W) \geq \begin{cases} -\frac{1}{1+\theta}, & \text{if } dI \leq a, \\ -1, & \text{if } dI > a. \end{cases} \tag{10}$$

Inequality (10) means that the marginal cost to investors of delivering the agent his continuation payoff, $-b'(W)$, can never exceed the cost of a lump-sum transfer in terms of investors' payoff for all W. Define $\widetilde{W}$ (or W^1) as the lowest value such that $b'(W) = -\frac{1}{1+\theta}$ (or $b'(W) = -1$) if $dI \leq a$ (or $dI > a$). Then, the optimal contract does not pay out cash $dI > 0$ until W_t exceeds the reflecting barrier $\widetilde{W}$ (or W^1) if $dI \leq a$ (or $dI > a$). Indeed, if $b'(W) > -\frac{1}{1+\theta}$ (or $b'(W) > -1$) when $dI \leq a$ (or $dI > a$), then promising one unit of continuation payoff to the agent costs the firm less than paying one unit of cash. As a result, investors hold the cash and promise to pay later.

The optimal payment policy for investors is represented in Proposition 1.

Proposition 1: *(i) Suppose that $\widetilde{W} + \theta a < W^1$. Then, it is optimal for investors to pay the agent according to*

$$dI = \begin{cases} 0, & \text{if } W \leq \widetilde{W}, \\ \frac{W-\widetilde{W}}{1+\theta}, & \text{if } \widetilde{W} < W \leq \widetilde{W} + (1+\theta)a, \\ a \cdot dt, & \text{if } \widetilde{W} + (1+\theta)a < W \leq W^1 + a, \\ W - W^1, & \text{if } W^1 + a < W. \end{cases}$$

(ii) Suppose that $W^1 \leq \widetilde{W} + \theta a$. Then, it is optimal for investors to pay the agent according to

$$dI = \begin{cases} 0, & \text{if } W \leq \widetilde{W}, \\ \frac{W-\widetilde{W}}{1+\theta}, & \text{if } \widetilde{W} < W \leq W^1 + a, \\ W - W^1, & \text{if } W^1 + a < W. \end{cases}$$

Proposition 1(i) shows that if $\widetilde{W} + \theta a < W^1$, the optimal contract consists of four segments in terms of the agent's continuation payoff W: for a range of sufficiently small W, no compensation is paid; thereafter compensation increases with W up to the reference point a; then there is a range of relatively high W in which compensation equals the reference point; and lastly there is a range of sufficiently high W in which compensation is increasing with W more rapidly. Proposition 1(ii) indicates that if $W^1 \leq \widetilde{W} + \theta a$, the optimal contract consists of three segments in terms of W. The difference is that in this case, there is no flat range where compensation is paid but is insensitive to W. However, in both cases, $\widetilde{W}$ (or $W^1 + a$) is a reflecting barrier where investors will pay the agent $\frac{W-\widetilde{W}}{1+\theta}$ (or $W - W^1$) to bring back W to $\widetilde{W}$ (or W^1) once $W \in (\widetilde{W}, \widetilde{W} + (1+\theta)a]$ in Proposition 1(i) and $W \in (\widetilde{W}, W^1 + a]$ in Proposition 1(ii) (or once $W \in (W^1 + a, \infty)$). In addition, irrespective of $\widetilde{W} + \theta a < W^1$ or $W^1 \leq \widetilde{W} + \theta a$, the sensitivity of dI to W is higher in the higher W range where dI increases with W than in the lower range. The intuition for the result of Proposition 1 is given after we derive Proposition 2.

In the rest of the analysis, we focus on the case of $\widetilde{W} + \theta a < W^1$. Note that $\widetilde{W} < W^1$ because we will prove that $b(W)$, consisting of $b_-(W)$ and $b_+(W)$ defined by (14) and (15) below, is concave for all $W \leq W^1 + a$ (see Proposition 2), and that $b'_-(\widetilde{W}) > b'_+(W^1)$ (see footnote 6). Thus, the inequality of $\widetilde{W} + \theta a < W^1$ always holds if θa is not very high. Then, we can assume without loss of generality that there will be an interval over which compensation is paid but is insensitive to W.

The payment policy given by Proposition 1 and the option to terminate keep the agent's continuation payoff between R and $W^1 + a$. Hence, it follows from (4), (8), and Proposition 1 that if $W \in [R, W^1 + a]$ and if the agent is telling the truth ($\widehat{Y}_t = Y_t$), the agent's continuation value evolves according to

$$\begin{cases} dW_t = \gamma W_t + \beta_t(dY_t - \mu dt) + \theta a dt, & \text{if } R \leq W_t \leq \widetilde{W}, \\ W_t = \widetilde{W}, & \text{if } \widetilde{W} < W_t \leq \widetilde{W} + (1+\theta)a, \\ dW_t = \gamma W_t - a dt + \beta_t(dY_t - \mu dt), & \text{if } \widetilde{W} + (1+\theta)a < W_t \leq W^1 + a. \end{cases} \tag{11}$$

We now need to characterize the investors' value function. Using Ito's lemma, it follows from (11) and Proposition 1 that the sum of the investors' expected cash flows and the expected change in the investors' value function are given by

$$E[dY_t - dI_t + db(W_t)] = \begin{cases} \mu dt + (\gamma W_t + \theta a)\, b'(W_t)dt + \frac{1}{2}\beta_t^2\sigma^2 b''(W_t)dt, \\ \quad \text{if } R \leq W_t \leq \widetilde{W}, \\ \mu dt - a dt + [\gamma W_t - a]\, b'(W_t)dt + \frac{1}{2}\beta_t^2\sigma^2 b''(W_t)dt, \\ \quad \text{if } \widetilde{W} + (1+\theta)a < W_t \leq W^1 + a. \end{cases} \tag{12}$$

Given Lemma 2, the agent's best response strategy is to report the truth if $\beta_t \geq \lambda[1 + \theta(dC_t, a)]$ for all $t \leq \tau$. Thus, it follows from (4) that the agent's best response strategy is to report the truth if $\beta_t(Y_t) \geq \lambda(1+\theta)$ for $dC_t \leq adt$, and if $\beta_t(Y_t) \geq \lambda$ for $dC_t > adt$. As the agent can choose any $dC_t \geq 0$ arbitrarily, this implies that his best response strategy is to report the truth if $\beta_t(Y_t) \geq \lambda(1+\theta)$. Because investors should earn an instantaneous total return equal to the discount rate r at the optimum, (12) implies that the Hamilton–Jacobi–Bellman equation for the investors' value function $b(W)$ consists of the following two parts:

$$rb_-(W) = \max_{\beta \geq \lambda(1+\theta)} \mu + (\gamma W + \theta a)\, b'_-(W) + \frac{1}{2}\beta^2\sigma^2 b''_-(W), \qquad \text{if } R \leq W \leq \widetilde{W}, \tag{13a}$$

$$rb_+(W) = \max_{\beta \geq \lambda(1+\theta)} \mu - a + (\gamma W - a)\, b'_+(W) + \frac{1}{2}\beta^2\sigma^2 b''_+(W), \quad \text{if } \widetilde{W} + (1+\theta)a < W \leq W^1 + a. \tag{13b}$$

Using the concavity of $b_-(W)$ and $b_+(W)$, as verified below, we must set $\beta = \lambda(1+\theta)$ in both cases. The investors' value function $b(W)$ thus satisfies the following second-order differential equations:

$$rb_-(W) = \mu + (\gamma W + \theta a)\, b'_-(W) + \frac{1}{2}\lambda^2(1+\theta)^2\sigma^2 b''_-(W), \qquad \text{if } R \leq W \leq \widetilde{W}, \tag{14a}$$

$$rb_+(W) = \mu - a + (\gamma W - a)\, b'_+(W) + \frac{1}{2}\lambda^2(1+\theta)^2\sigma^2 b''_+(W), \quad \text{if } \widetilde{W} + (1+\theta)a < W \leq W^1 + a, \tag{14b}$$

with

$$b_-(W) = b_-(\widetilde{W}) - \frac{W - \widetilde{W}}{1+\theta}, \qquad \text{if } \widetilde{W} < W \leq \widetilde{W} + (1+\theta)a, \tag{15a}$$

$$b_+(W) = b_+(W^1 + a) - (W - W^1 - a), \qquad \text{if } W^1 + a < W. \tag{15b}$$

Note that (15b) is derived from $[b_+(W^1) - b_+(W^1 + a)] + (W - W^1 - a) + b_+(W) = b_+(W^1)$ for $W \in (W^1 + a, \infty)$.

Termination delivers a boundary condition at $W = R$. In addition, investors are indifferent between paying $dI = \frac{W-\widetilde{W}}{1+\theta}$ and $dI = a$ to the agent at $W = \widetilde{W}$ $+ (1 + \theta)a$. This condition provides another boundary condition. The optimality of the cash payment also yields four other boundary conditions. These six boundary conditions yield a solution to equation (14) and the boundaries $\widetilde{W}$ and $W^1 + a$. First, investors terminate the contract when the agent's continuation value becomes equal to his reservation level, $b_-(R) = L$. Second, paying $dI = \frac{W-\widetilde{W}}{1+\theta}$ to the agent brings about the same level of investors' value function as paying $dI = a$ to the agent. Thus, $b_-(W)$ and $b_+(W)$ must be connected smoothly. Given (15a), this implies that $b_-(\widetilde{W}+(1+\theta)a) = b_-(\widetilde{W})$ $- a = b_+(\widetilde{W} + (1+\theta)a)$. Third, for the optimality of the cash payment, we need the smooth pasting conditions: the first derivatives must agree at the boundaries $\widetilde{W}$ and $W^1 + a$, $b'_-(\widetilde{W}) = -\frac{1}{1+\theta}$ and $b'_+(W^1 + a) = -1$.[6] We also need to have the super contract conditions: the second derivatives must agree at the boundaries $\widetilde{W}$ and $W^1 + a$, $b''_-(\widetilde{W}) = 0$ and $b''_+(W^1 + a) = 0$. It follows from (14), $b'_-(\widetilde{W}) = -\frac{1}{1+\theta}$, $b'_+(W^1 + a) = -1$, $b''_-(\widetilde{W}) = 0$, and $b''_+(W^1 + a) =$ 0 that the super contract conditions are rewritten as

$$rb_-(\widetilde{W}) + \frac{\gamma\widetilde{W} + \theta a}{1+\theta} = \mu, \tag{16}$$

$$rb_+(W^1 + a) + \gamma(W^1 + a) = \mu. \tag{17}$$

Figure 1 shows an example of the investors' value function $b(W)$, which is given by $b_-(W)$, consisting of the curve AB and the line BC with the slope $-\frac{1}{1+\theta}$, and $b_+(W)$ consisting of the curve CE and the attached line with slope -1 for $W > W^1 + a$.

To conclude this subsection, we compare equation (14) with the corresponding equation in DeMarzo and Sannikov (2006), which can be derived by setting

[6] It follows from Proposition 1 that the agent stays in the gain space as long as $W^1 + a < W$. Because the first derivative of the investors' value function must be equal to -1 in the gain space as long as payments to the agent are positive, we must have $b'_+(W^1 + a) = -1$. In addition, as the agent remains in the loss space at $W = W^1$ under the optimal contract, we must also obtain $-1 < b'_+(W^1) < -\frac{1}{1+\theta}$. Although W^1 is defined as the lowest value such that $b'(W^1) = -1$ if $dI > a$, note that W^1 belongs to the range of the loss space (dI $\leq a$) under the optimal contract. Hence, $b'_+(W^1)$ cannot be equal to -1.

$a = \theta = 0$ in our model. Hence, their evolving equation of $b(W)$ is expressed as

$$rb(W) = \mu + \gamma W b'(W) + \frac{1}{2}\lambda^2\sigma^2 b''(W), \qquad \text{if } R \le W \le W^1, \tag{18}$$

with $b(R) = L$, $b'(W^1) = -1$, and $rb(W^1) + \gamma W^1 = \mu$.[7] Thus, the first noticeable difference is that the dynamic system consists of two second-order differential equations ((14a) and (14b)) and two payment thresholds ($\widetilde{W}$ and $W^1 + a$) in our model with loss aversion, whereas it consists of one second-order differential equation and one payment threshold (W^1) in DeMarzo and Sannikov (2006). The second key difference is found in the coefficients of $b'(W)$ and $b''(W)$. In particular, the coefficient of $b''(W)$ is $\frac{1}{2}\lambda^2(1+\theta)^2\sigma^2$ in our model, whereas it is $\frac{1}{2}\lambda^2\sigma^2$ in DeMarzo and Sannikov (2006). Because the coefficient of $b''(W)$ corresponds to the diffusion part of the state variable and diffusion captures incentives, the presence of loss aversion requires greater incentives along the optimal path.

3.1.3. The optimal contract The following proposition formalizes our findings about the optimal contract that are derived in Section 3.1.2. The verification argument is also given in the proof of this proposition.

Proposition 2: *(i) Under the optimal contract, the investors' continuation payoff $b(W)$ consists of two parts $b_-(W)$ for $W \in [R, \widetilde{W} + (1+\theta)a]$ and $b_+(W)$ for $W \in (\widetilde{W} + (1+\theta)a, \infty)$, and satisfies (14) and (15) with boundary conditions $b_-(R) = L$, $b_-(\widetilde{W}) - a = b_+(\widetilde{W} + (1+\theta)a)$, $b'_-(\widetilde{W}) = -\frac{1}{1+\theta}$, $b'_+(W^1 + a) = -1$, (16) and (17). The function $b(W)$ is concave on $W \in [R, W^1 + a]$; in particular, $b_-(W)$ is strictly concave on $W \in [R, \widetilde{W})$ and $b_+(W)$ is strictly concave on $W \in (\widetilde{W} + (1+\theta)a, W^1 + a)$.*

(ii) The agent's continuation payoff W_t evolves according to (11). When $W_t \in [R, \widetilde{W}]$, $dI_t = 0$. When $W_t \in (\widetilde{W}, \widetilde{W} + (1+\theta)a]$, a payment $\frac{W_t - \widetilde{W}}{1+\theta}$ is made. When $W_t \in (\widetilde{W} + (1+\theta)a, W^1 + a]$, $dI_t = a \cdot dt$. When $W^1 + a$

[7] Note that $\widetilde{W} = W^1 = W^1 + a$ and $b'_-(\widetilde{W}) = b'_+(W^1 + a) = b'_+(W^1) = -1$ when $a = \theta = 0$.

$< W_t$, *a payment* $W_t - W^1$ *is made. When* $W_t = \widetilde{W}$, *payments* dI_t *cause* W_t *to reflect at* $\widetilde{W}$ *as long as* $W_t \in (\widetilde{W}, \widetilde{W} + (1 + \theta)a]$. *When* $W_t = W^1 + a$, *payments* dI_t *cause* W_t *to reflect at* W^1 as long as $W_t \in (W^1 + a, \infty)$. *Thus,* $\widetilde{W}$ and W^1 are absorbing states. *When* W_t *falls to* R, *the contract is terminated at time* τ.

Corollary to Proposition 2: *(i) The optimal payment schedule has two flat segments:* $dI_t = 0$ *for* $W_t \in [R, \widetilde{W}]$ and $dI_t = a$ *for* $W_t \in (\widetilde{W} + (1 + \theta)a, W^1 + a]$. *For* $W_t \in (\widetilde{W}, \widetilde{W} + (1 + \theta)a]$ *and* $W_t \in (W^1 + a, \infty)$, *payment is linearly increasing in the agent's continuation value. These segments are connected continuously.*
(ii) In the continuous-time agency model with cash diversion, investors provide stronger incentives to the agent along the optimal path in the presence of loss aversion than in its absence. In addition, the introduction of loss aversion induces investors to reward the agent earlier in the sense that investors start paying the agent cash even though the investors and agent's required expected returns do not exceed the available expected cash flows.

Comparing the optimal contract given by Proposition 2 with that given by DeMarzo and Sannikov (2006, Proposition 1), we can show how loss aversion affects the features of the optimal contract. First, the introduction of loss aversion induces investors to reward the agent earlier. In our model, investors start paying the agent cash when W_t reaches $\widetilde{W}$ before reaching $W^1 + a$, which satisfies (17). In DeMarzo and Sannikov (2006), the principal postpones payment to the agent by $W^{1,DS}$, which is determined by $rb(W^{1,DS}) + \gamma \cdot W^{1,DS} = \mu$, where $b(W)$ satisfies (18).[8] This suggests that the introduction of loss aversion induces investors to start paying the agent cash before the investors and agent's required expected returns exhaust the available expected cash flows.[9]

[8] To distinguish between W^1 in DeMarzo and Sannikov (2006) and our W^1, we denote the former by $W^{1,DS}$.

[9] Note that $rb_-(W_t) + \gamma W_t < \mu$ for all $W_t \in [R, \widetilde{W} + (1 + \theta)a]$ and $rb_+(W_t) + \gamma W_t < \mu$ for all $W_t \in (\widetilde{W} + (1 + \theta)a, W^1 + a)$ because $\gamma > r$, $b'_-(W) \geq -\frac{1}{1+\theta}$ for all $W \in [R, \widetilde{W} +$

Intuitively, the agent's impatience ($\gamma > r$) implies that the optimal contract pays cash to the agent as early as possible. However, paying cash to the agent earlier reduces the agent's continuation payoff. Under limited liability of the agent, investors are forced to terminate the contract when the agent's continuation payoff becomes R. Thus, paying cash to the agent earlier might increase the likelihood of future inefficient contract termination. As a result, the optimal contract requires investors to set the optimal cash payment boundary so that the marginal benefit and the marginal cost of investors paying cash to the agent are equal. In fact, the marginal benefit of investors paying cash to the agent is greater in the presence of loss aversion than in its absence because the positive payment to the agent reduces disutility from loss aversion in the loss space by $\lambda\theta$. This is because the marginal benefit of investors paying cash is raised discontinuously within the loss space when loss aversion (the higher sensitivity to losses than to gains around the reference point) applies. Hence, the effect of loss aversion induces investors to pay cash earlier.

Second, an increase in the agent's degree of loss aversion forces investors to provide the agent with a stronger incentive. More precisely, in our model, the sensitivity of W_t with respect to the agent's report, β_t, equals the sum of the magnitude of the agency problem, λ, and the degree of loss aversion, θ, multiplied by λ (that is, $\beta_t = \lambda(1 + \theta)$). By contrast, in DeMarzo and Sannikov (2006), the sensitivity just equals λ .

The intuition behind this result is that to avoid inefficiency resulting from liquidation, investors can reduce the risk involved in lowering W_t; that is, investors can reduce the probability that W_t reaches R, thus lowering the probability of costly liquidation. To reduce the probability that W_t reaches R, it is optimal for investors to make the sensitivity of W_t with respect to the agent's report, β_t, as low as possible, provided that the level of sensitivity does not violate his incentives to tell the truth. This implies that β_t must be equal

$(1 + \theta)a]$, $-\frac{1}{1+\theta} > b'_+(W) > -1$ for all $W \in (\widetilde{W} + (1 + \theta)a,\, W^1 + a)$, $b_-(\widetilde{W} + (1 + \theta)a) = b_+(\widetilde{W} + (1 + \theta)a)$, and (17) hold. Similarly, we can prove that $rb(W_t) + \gamma W_t < \mu$ for all $W_t \in [R,\, W^{1,DS})$ in DeMarzo and Sannikov (2006).

to $\lambda(1 + \theta)$. Indeed, underreporting income by one unit increases consumption by λ and decreases the agent's disutility from loss aversion in the loss space by $\lambda\theta$, whereas it reduces the agent's continuation payoff by β_t. Hence, if $\beta_t = \lambda(1 + \theta)$, this sensitivity gives the agent just enough incentive to report a true realization of income. However, in the absence of loss aversion (that is, $\theta = 0$), investors only need to set $\beta_t = \lambda$.

This result is in contrast with the result in Herweg, Müller, and Weinschenk (2010) that an increase in the agent's degree of loss aversion may allow the principal to use weaker incentives. In their static model, loss aversion may be associated with a strong incentive for the agent to choose a high effort because the high effort can affect the probability distribution of the outcome and thereby reduce the scope for incurring a loss. By contrast, in our model, loss aversion can increase the gains of cash diversion because cash diversion can increase the agent's consumption and thereby reduce the scope for incurring a loss. Hence, to prevent the agent from diverting cash flows, investors need to provide a stronger incentive for the agent under loss aversion. This problem is discussed further in Section 3.1.4.

Third, loss aversion implies that, unlike in DeMarzo and Sannikov (2006), the agent's compensation may be insensitive to his continuation payoff over some intervals even after investors start paying the agent cash. In the loss-aversion literature, it is common that there is a range in which payment does not vary with performance (see de Meza and Webb (2007) and Herweg, Müller, and Weinschenk (2010)). If the agent's continuation payoff can be interpreted as a kind of performance,[10] our result also verifies the existence of a performance-independent flat part in the range of intermediate performance. Furthermore, in our model, for a smaller W, there exists a payment threshold, $\widetilde{W}$, as an absorbing state below which investors postpone payment to the agent and to which W_t returns when $W_t \in (\widetilde{W}, \widetilde{W} + (1 + \theta)a]$. For a higher W, there also exists another payment threshold, $W^1 + a$, below which

[10]Indeed, W can be viewed as a proxy for equity value to outside investors. See the arguments in Section 5.

the agent's compensation is positive but constant for $W_t \geq \widetilde{W} + (1+\theta)a$, and another absorbing state, W^1, to which W_t returns when $W_t \in (W^1 + a, \infty)$.[11] The sensitivity of the agent's compensation is higher when $W_t \in (W^1 + a, \infty)$ than when $W_t \in (\widetilde{W}, \widetilde{W} + (1+\theta)a]$. By contrast, in DeMarzo and Sannikov (2006), there exists only one payment threshold as an absorbing state below which investors postpone payment to the agent.

Intuitively, as the agent is more sensitive to losses than gains at the reference point, loss aversion causes a discontinuous drop in the agent's marginal payoff when consumption is below the reference point. This implies that if the expected payoff to the agent needs to be maintained, expected payments in the gain space must increase by proportionally more than the same amount of reductions in the expected payments in the loss space; that is, entering the loss space is an expensive way to compensate the agent. Hence, investors have an incentive to increase payments to the agent in the loss space in order to reduce the scope for the agent to incur a loss or to reduce the agent's disutility from loss aversion in the loss space. As a result, when $W_t \in (\widetilde{W}, \widetilde{W} + (1+\theta)a]$, investors make the cash payment $\frac{W_t - \widetilde{W}}{1+\theta}$ to the agent immediately and maintain W_t at $\widetilde{W}$. However, for $\widetilde{W} + (1+\theta)a < W_t$, if investors make the cash payment $\frac{W_t - \widetilde{W}}{1+\theta}$ immediately, the cash payment leaves the agent in the gain space. Because loss aversion does not operate in the gain space and investors want to avoid inefficient contract termination, investors have no incentive to make the cash payment $\frac{W_t - \widetilde{W}}{1+\theta}$ in the gain space as long as W_t is not sufficiently large. Instead, investors make payments exactly equal to the reference level of income, a, so that they reduce the agent's disutility from loss aversion as much as possible but avoid leaving the agent in the gain space when $W_t \in (\widetilde{W} + (1+\theta)a, W^1 + a]$. When $W^1 + a < W_t$, investors start making a cash payment to the agent of $W_t - W^1$ immediately and maintain W_t at W^1 because they now have an incentive to do so even in the absence of

[11] In fact, it follows from Proposition 2 that the payment $dI_t = a$ (or $dI_t = W_t - W^1$) causes W_t to return to W^1 when $W_t \in (W^1, W^1 + a]$ (or when $W_t \in (W^1 + a, \infty)$). As a result, we can also state that W_t returns to W^1 when $W_t \in (W^1, \infty)$.

loss aversion. The reason is that the possibility of inefficient contract termination is sufficiently small for such a sufficiently high level of W so that the marginal cost of investors paying cash to the agent becomes sufficiently small. The sensitivity of the agent's compensation to his continuation payoff is higher in this range because the benefit to the agent of receiving cash is greater in the absence of loss aversion.

The intuition for why there are two absorbing states, $\widetilde{W}$ and W^1, is because of the formulation that loss aversion implies higher sensitivity to losses than to gains around the reference point. Hence, there exists one absorbing state corresponding to the loss space and another absorbing state corresponding to the gain space.

3.1.4. Further discussion The introduction of loss aversion into the continuous-time agency model à la DeMarzo and Sannikov (2006) has three significant effects on the optimal contract. First, the presence of loss aversion weakens the agent's incentive to report truthfully because diverting cash flows reduces his disutility from loss aversion. Hence, to satisfy the incentive-compatibility constraint for the agent, the sensitivity parameter of his continuation payoff toward his report must increase and hence the volatility of his continuation payoff increases. This implies that the introduction of loss aversion requires stronger incentives along the optimal path. Furthermore, this result is not restricted to the framework of the cash-diversion model. As long as the agent's hidden effort directly affects both the firm's cash flow and his consumption, we can show that this result still holds in a standard hidden effort model, using an analysis similar to that of DeMarzo and Sannikov (2006, Section III). By contrast, in Herweg, Müller, and Weinschenk (2010), the presence of loss aversion gives the agent more incentive to reduce the probability of entering the loss space. Hence, this possibility enables the principal to use weaker incentives under loss aversion. One reason for this difference is that in the model of Herweg, Müller, and Weinschenk (2010), loss aversion induces the agent to exert more effort and to avoid the high probability of a low outcome because his effort can affect the probability distribution of the outcome. Thus, in their model, loss aversion can mitigate the incentive-compatibility constraint. In fact, in the static moral hazard model of Dittmann, Maug, and Spalt (2010) where the agent's effort does not affect the probability distribution of the outcome, the calibration results show that an increase in the agent's degree of loss aversion may induce stronger incentives.

Second, the introduction of loss aversion also affects when and how much the agent receives in compensation in order to reduce the cost to investors. In our continuous-time agency model, the agent's incentive is generated only through a variation in his continuation payoff. Thus, the payment to the agent has no immediate effect on his incentive to report truthfully. This implies that the problem of how the agent is given incentives can be separated from the problem of when and how much compensation he receives according to the level of W_t. This is in contrast with the standard literature of loss aversion, in which the incentive consideration strongly affects the payment schedule to the agent. Instead, in our model, the payment schedule to the agent depends mainly on three factors: the agent's degree of loss aversion, the agent's impatience, and the possibility of inefficient future liquidation. The first two factors relate to the marginal benefit to investors of paying cash earlier, whereas the final factor is concerned with the marginal cost to investors of paying cash earlier. The optimal contract then requires investors to set the optimal cash payment so that the marginal benefit equals the marginal cost.

Third, our result regarding the optimal cash payment indicates that in the presence of loss aversion, there are more ranges of W_t in which the agent is rewarded with positive compensation. Furthermore, in these ranges, the optimal compensation schedule is of two types: payment is insensitive to W_t or payment is increasing proportionately in W_t. More specifically, the optimal compensation schedule not only explains the existence of a flat part insensitive to W_t when W_t is in the intermediate range, but also shows that the flat part is preceded and followed by a range of option-type payoffs where the cash payment is increasing in W_t and causes W_t to stay at the absorbing states.[12] Option-type contracts are also rationalized in the static contract model of loss aversion in de Meza and Webb (2007) and Dittmann, Maug, and Spalt (2010). De Meza and Webb (2007) obtain option-type contracts by assuming that the agent is risk averse in the loss space. Dittmann, Maug, and Spalt (2010) discuss the optimality of option-type contracts, but their result depends on the feature that the optimal contract is convex in the gain space because risk tolerance increases quickly when the distance from the reference point increases.[13] By contrast, in our model, the optimality of the option-type contract does not depend on the assumption of the agent's risk aversion or risk tolerance. Rather, option-type contracts are derived through the interaction between the agent's loss aversion, the agent's impatience, and the possibility of inefficient future liquidation. Furthermore, we show that in the presence of loss aversion, the sensitivity of the agent's compensation to W_t is higher in the higher range of W_t with option-type payoffs than in the lower range.

3.2. Hidden Savings.—

We now relax our assumptions by allowing the agent to save secretly and to steal at an unbounded rate. In fact, applying a procedure similar to that of DeMarzo and Sannikov (2006), we can prove that even in this case, the agent has an incentive to report truthfully and maintain zero savings even though he can save within the firm without paying the diversion cost or can save in his own account by paying the diversion cost.

Proposition 3: *Suppose that the process W_t is bounded from above and satisfies*

$$dW_t = \gamma W_t dt - dI_t + \lambda[1 + \theta(dC_t, a)](d\widehat{Y}_t - \mu dt) - \theta(dC_t, a)(dC_t - adt), \quad (19)$$

[12] As mentioned above, W_t can be interpreted broadly as a proxy for equity value to outside equity holders.

[13] Risk tolerance is defined by the inverse of absolute risk aversion. It is cheaper for the principal to provide incentives in regions where risk tolerance is high.

until stopping time $\tau = \min\{t \mid W_t = R\}$. If the agent cannot overreport by putting his own money back into the project, then he receives a payoff of at most W_0 from any feasible strategy in response to a contract (τ, I). In addition, the payoff W_0 is attained if the agent reports truthfully and maintains zero savings.

Proposition 3 shows that the optimal contract given by Proposition 2 remains incentive compatible, even though the agent can save secretly and steal at an unbounded rate, if the agent cannot overreport by putting his own money back into the project. The intuition is that under risk neutrality, the marginal benefit to the agent of reporting or consuming cash is constant over time if he continues to stay in the loss or gain space, and that the marginal benefit is decreasing if the agent moves from the gain space to the loss space. As private savings grow at rate $\rho < \gamma$, there is no incentive to save secretly or to misrepresent income because misreporting delays consumption and may leave the agent in the loss space in the early period if the agent cannot overreport.

4. Implementation of the Optimal Contract

The optimal contract we have derived can be implemented and interpreted readily in terms of standard securities that include equity, long-term debt, and a line of credit. These securities are held by widely dispersed investors or intermediaries. As the core results describing the contracts have been given in the preceding section, we merely reinterpret these results in this section.[14]

The firm raises initial capital K and possibly additional cash in an optimal contract. The optimal contract is implemented by issuing securities at time 0. The securities used in the implementation are the same as those in DeMarzo and Sannikov (2006). More specifically, the agent holds a fraction α of the firm's equity. The remaining fraction of the firm's equity is held by outside investors. Equity holders receive dividend payments paid from the firm's avail-

[14] The optimal contract is written conditional on the agent's continuation payoff W. Thus, the implementation result is unaffected regardless of whether the agent designs the securities to maximize his own payoff or investors design the securities to maximize the value of the firm.

able cash or credit. However, the agent does not receive part of the liquidation payoff. In this sense, the agent's equity is inside equity with the provision that it is worthless in the event of termination. Outside investors also hold long-term debt and a line of credit. Long-term debt is a consol that pays continuous coupons at rate x and has the face value D.[15] We let the coupon rate be r, so that $D = \frac{x}{r}$. If the firm defaults on a coupon payment, debt holders force termination of the project. A revolving line of credit provides the firm with available credit up to a limit C^L. Balances on the line of credit are charged a fixed interest rate r^c. The firm borrows and repays funds on the line of credit at the discretion of the agent. If the balance on the line of credit exceeds C^L, the firm defaults and the project is terminated.

The next proposition shows that the optimal contract can be implemented with a capital structure based on the securities introduced above.[16]

Proposition 4: *If $\lambda(1+\theta) \leq 1$, there exists a capital structure that implements the optimal contract and has the following features:*

$$\alpha = \lambda(1+\theta), \tag{20a}$$

$$rD_t = \begin{cases} \mu - \frac{\gamma R}{\lambda(1+\theta)} - \gamma C^L - \frac{\theta a}{\lambda(1+\theta)}, & \text{if } M_{\widetilde{W}} \leq M_t, \\ \mu - \frac{\gamma R}{\lambda(1+\theta)} - \gamma C^L - \frac{\theta\{(1+\theta)a - [R + \lambda(1+\theta)(C^L - M_t) - \widetilde{W}]\}}{\lambda(1+\theta)^2}, & \text{if } M_{\widetilde{W}+(1+\theta)a} \leq M_t < M_{\widetilde{W}}, \\ \mu - \frac{\gamma R}{\lambda(1+\theta)} - \gamma C^L, & \text{if } 0 \leq M_t < M_{\widetilde{W}+(1+\theta)a}, \end{cases} \tag{20b}$$

$$C^L = \frac{W^1 + a - R}{\lambda(1+\theta)}, \tag{20c}$$

where M_t is the credit line balance, $M_{\widetilde{W}} = C^L - \frac{\widetilde{W}-R}{\lambda(1+\theta)}$, and $M_{\widetilde{W}+(1+\theta)a} = C^L - \frac{\widetilde{W}+(1+\theta)a-R}{\lambda(1+\theta)}$. The line of credit has interest rate $r^C = \gamma$. For the balance $M_t \geq 0$, the agent's continuation payoff W_t is determined by the current draw

[15] If $D < 0$, long-term debt is interpreted as a compensating balance, as in DeMarzo and Sannikov (2006).

[16] In this capital structure implementation, the agent can choose when to draw on or repay the credit line, how much to pay in dividends, and whether to accumulate cash balances within the firm.

M_t on the line of credit:

$$W_t = R + \lambda(1+\theta)(C^L - M_t). \tag{21}$$

Furthermore, dividends are also paid according to the current draw M_t:

$$dDiv_t = \begin{cases} 0, & \text{if } M_{\widetilde{W}} \le M_t, \\ \frac{R+\lambda(1+\theta)(C^L - M_t)-\widetilde{W}}{\lambda(1+\theta)^2}, & \text{if } M_{\widetilde{W}+(1+\theta)a} \le M_t < M_{\widetilde{W}}, \\ \frac{a}{\lambda(1+\theta)}, & \text{if } 0 < M_t < M_{\widetilde{W}+(1+\theta)a}. \end{cases} \tag{22}$$

Once the line of credit is fully repaid, all excess cash flows are issued to the agent as dividends or stock options that are bought back by the firm immediately.

Comparing the result of Proposition 4 with that in DeMarzo and Sannikov (2006, Proposition 3), we can explain how the degree of loss aversion affects the implementation procedure. First, (20a) shows that in order to eliminate the agent's incentive to divert cash, investors need to provide the agent with a fraction of equity $\lambda(1+\theta)$, which is larger than that in their model. That is, the agent's equity holding ratio is increasing in λ and θ in our model, whereas it is increasing only in λ in DeMarzo and Sannikov (2006). The intuition for this difference is that in our model, underreporting income by one unit not only increases consumption by λ but also reduces the agent's disutility from loss aversion in the loss space by $\lambda\theta$. Thus, with a higher λ or θ, investors must increase the agent's equity holding ratio in order to provide the agent with adequate incentive to report a true realization of income.[17] In addition, if $\lambda(1+\theta) > 1$, the agent's share of equity exceeds 1. Hence, the implementation

[17] In the static moral hazard model with loss aversion of Herweg, Müller, and Weinschenk (2010), the principal uses weaker incentives when θ increases. In our context, this implies that the agent's equity holding ratio is decreasing in θ. Such a difference stems from the fact that in their model, loss aversion is associated with a strong incentive for the agent to choose a high effort to reduce the scope for incurring a loss. Hence, unlike our model, loss aversion reduces the need for investors to give the agent more equity grants in order to induce him to choose a high effort.

procedure given by Proposition 4 is impossible if $\lambda(1+\theta) > 1$.

Second, as in DeMarzo and Sannikov (2006), equation (21) ensures that the agent does not pay dividends prematurely by drawing down the line of credit $C^L - M$ immediately and then defaulting. This is because (21) implies that the sum of the agent's immediate payoff, the reduction of his disutility from loss aversion, and his termination payoff when he follows this deviation—the right-hand side of (21)—would be equal to W_t, which he can receive by committing to the rule of our capital structure implementation.

Third, as long as $\theta > 0$, (20b) shows that long-term debt is larger in our model than in DeMarzo and Sannikov (2006, equation (17)) if the credit line balance is sufficiently low ($0 \le M_t < M_{\widetilde{W}+(1+\theta)a}$). However, if the credit line balance is not sufficiently low ($M_{\widetilde{W}+(1+\theta)a} \le M_t$), we cannot determine unambiguously whether long-term debt is larger in our model than in their model. In addition, unlike their model, long-term debt is decreasing in M_t if the credit line balance is in the intermediate range ($M_{\widetilde{W}+(1+\theta)a} \le M_t < M_{\widetilde{W}}$). Intuitively, for a given M_t, the role of long-term debt is to adjust the profit rate of the firm so that W_t satisfies (21). Indeed, if M_t is not sufficiently low, the agent's compensation is less than the reference point. Thus, long-term debt can be reduced by an amount that is proportional to the agent's disutility from loss aversion. By contrast, if M_t is sufficiently low, the agent's compensation equals the reference point. As the agent's disutility from loss aversion is equal to zero, long-term debt need not be adjusted by his disutility from loss aversion in this case. However, under loss aversion, W_t must increase to satisfy (21) relative to M_t because the sensitivity of W_t with respect to the agent's report increases up to $\lambda(1+\theta)$. Hence, long-term debt becomes larger in our model than in their model if M_t is sufficiently low. In the intermediate range of M_t, long-term debt can be decreasing in M_t because dividend payments are decreasing in M_t, as argued below.

Fourth, in our model, dividends are paid as long as the credit line balance is $M_t < M_{\widetilde{W}}$, whereas in DeMarzo and Sannikov (2006), dividends are paid only when the credit line balance is $M_t = 0$. Furthermore, in our model, if the credit line balance is sufficiently low, dividend payments are positive but independent of M_t (and W_t), that is, insensitive to the firm's performance; if the credit line balance is intermediate, dividend payments are decreasing in M_t; and if the credit line balance is sufficiently large, no dividends are paid. Thus, overall, dividend payments are nonincreasing in M_t. Hence, a large line of credit delays or reduces the agent's consumption, but also provides the project with more flexibility by delaying termination. The intuition is that loss aversion induces investors to pay cash earlier in our optimal contract. Thus, dividends must be paid before the credit line balance decreases to zero. Furthermore, if M_t increases, then W_t must decrease in order to satisfy the incentive-compatibility constraint of our capital structure, (21). Given that the agent's compensation is increasing in W_t for $W_t \in (\widetilde{W}, \widetilde{W} + (1+\theta)a]$ (or independent of W_t for $W_t \in [R, \widetilde{W}] \cup (\widetilde{W} + (1+\theta)a, W^1 + a))$ and that the agent's compensation is paid as dividends in this capital structure, dividend payments must be decreasing in M_t for $M_t \in [M_{\widetilde{W}+(1+\theta)a}, M_{\widetilde{W}})$ (or independent of M_t for $M_t \in (0, M_{\widetilde{W}+(1+\theta)a}) \cup [M_{\widetilde{W}}, \infty))$.

Finally, if γ is close to r, the total debt capacity of the firm is represented by

$$D_t + C^L \simeq \begin{cases} \frac{\mu}{\gamma} - \frac{R}{\lambda(1+\theta)} - \frac{\theta a}{\gamma\lambda(1+\theta)}, & \text{if } M_{\widetilde{W}} \leq M_t, \\ \frac{\mu}{\gamma} - \frac{R}{\lambda(1+\theta)} - \frac{\theta\left\{(1+\theta)a - \left[R + \lambda(1+\theta)(C^L - M_t) - \widetilde{W}\right]\right\}}{\gamma\lambda(1+\theta)^2}, & \text{if } M_{\widetilde{W}+(1+\theta)a} \leq M_t < M_{\widetilde{W}}, \\ \frac{\mu}{\gamma} - \frac{R}{\lambda(1+\theta)}, & \text{if } 0 \leq M_t < M_{\widetilde{W}+(1+\theta)a}. \end{cases} \tag{23}$$

As the right-hand side of (23) depends on M_t in the intermediate range of M_t, the total debt capacity of the firm is sensitive to the volatility, σ, and the liquidation value, L, unlike in DeMarzo and Sannikov (2006).

5. Empirical Implications

In this section, we discuss the empirical implications of our model with respect to the following issues: (i) the evolution of CEO compensation and the low level of stability of CEOs' fractional equity ownership after World War II in the United States, and (ii) corporate dividend-smoothing policy.

We begin by providing empirical implications for the first issue. Broadly speaking, after World War II, the evolution of CEO compensation can be divided into three distinct periods (see Frydman and Jenter (2010) and Frydman and Saks (2010)). Before the 1970s, low levels of pay, little dispersion across top managers, and moderate pay–performance sensitivities were observed. From the mid-1970s to the end of the 1990s, compensation levels trended upward dramatically, differences in compensation across managers and firms increased, and, in particular, stock options grew substantially to become the single largest component of CEO compensation in the 1990s. The sensitivity of CEO wealth to firm performance also surged in the 1990s, mostly because of rapidly growing option portfolios. During the 2000s, average CEO compensation declined, and restricted stock grants replaced stock options as the most popular form of stock compensation. However, most CEOs' fractional equity ownership remained low throughout these three periods, although it has increased slowly.

There are two prominent theories for explaining the evolution of CEO compensation. One is the managerial power (rent extraction) theory in which the high level of CEO compensation is the result of executives' ability to set their own pay and extract rents from the firms they manage (see Bebchuk and Fried (2003, 2004)). The other is the competitive pay theory in which the high level of CEO compensation is viewed as the efficient outcome of a managerial labor market where firms optimally compete for managerial effort or talent (for a review of the literature, see Frydman and Jenter (2010)). However, in light of the evidence that corporate governance has strengthened considerably over the past 30 years (see Holmstrom and Kaplan (2001), Hermalin (2005), and Kaplan (2008)), the managerial power theory cannot explain the steady increase in CEO compensation since the 1970s. Although market-based and optimal contracting explanations for CEO pay have the predicted effects, the estimated magnitudes are modest, and so they leave much of the sharp rise in CEO compensation unexplained (for a discussion, see Frydman and Jenter (2010)). Thus, none of these theories provides a fully convincing explanation for the evolution of CEO compensation since the 1970s. In particular, these hypotheses fail to explain the explosive growth of options in the 1990s and the recent replacement of options by restricted stock, even while most CEOs' fractional equity ownership remained low in these periods.

In our model, equity holders receive total dividends of $dDiv_t = dI_t/\lambda(1+\theta)$ per share because the agent receives a fraction $\lambda(1+\theta)$. At contract termination, outside equity holders also receive the remaining part of the liquidation value, $L_E = \max(0, L - D - C^L)/[1-\lambda(1+\theta)]$, per share after the debt and the line of credit have been paid off. Then, the per share value of equity to outside equity holders is represented by $V_E(W) = E\left[\int_0^{\tau} e^{-rs} dDiv_s + e^{-r\tau} L_E \middle| W\right]$. If the liquidation value received by outside equity holders is sufficiently small at termination, $V_E(W)$ is nondecreasing in W because $dDiv$ or dI is nondecreasing in W. Thus, W can be viewed broadly as a proxy for equity value to outside equity holders.

Then, the optimal compensation policy given by Proposition 2 in our model can be interpreted literally as an option-like scheme. If W is sufficiently small, the agent is paid nothing. If W is in the middle range, the agent is paid by salary and bonus or by dividends from restricted stock; however, in the case of relatively small W in this range, his compensation is reduced according to W when W decreases. If W is sufficiently large, the agent additionally receives option grants and exercises them immediately;[18] furthermore, pay–performance sensitivity is higher in this range of W than in any other range of

[18] These option grants may include performance-based vesting provisions that allow managers to cash in the vested stock/options once they achieve a certain performance target (see Bettis, Bizjak, Coles and Kalpathy (2010)).

W. Hence, in our model, the evolution of CEO compensation can be explained by the evolution of W, that is, equity value to outside investors. Before the 1970s, equity values remained low. From the mid-1970s to the end of the 1990s, equity values grew rapidly; the result was explosive growth in option grants. During the 2000s, equity values declined, as did the use of option grants.[19]

In addition, in our model, the agent holds a fraction of equity, $\lambda(1 + \theta)$. Suppose that an increase in firm size, a complicated change in technology or a higher volatility of the business environment gradually increases the magnitude of the agency problem λ or the degree of loss aversion θ. Then, these changes gradually induce larger firms, more technology-oriented firms or firms facing more competitive and riskier environments to grant more equity to their managers. If these changes occur gradually over time, most CEOs' fractional equity ownership will remain low in these periods. Thus, our model provides explanations for not only the evolution of CEO compensation after World War II but also the low fractional ownership levels of most CEOs in the same period, even though CEOs' fractional equity ownership is gradually increasing.

For the evolution of dividends, our theory also derives new implications of why firms smooth dividends or what determines a firm's propensity to smooth dividends. Dividend smoothing is one of the most widely documented phenomena in corporate financing policy, because firms' primary concern is the stability of dividends (see Leary and Michaely (2011)). In the past two decades, stock repurchases have been used increasingly in place of dividends (see Skinner (2008)). Hence, when we derive implications from our results, we may interpret $dDiv$ as the total payout policy that includes dividends and stock repurchases, although the total payout is significantly less smoothed than dividends (see Skinner (2008) and Leary and Michaely (2011)).

Now, our result of the evolution of $dDiv$ given by (22) shows that there is an interval over which $dDiv$ is actually paid and is smoothed. This smoothing

[19] It is difficult to explain the evolution of CEO compensation within the framework of DeMarzo and Sannikov (2006) because in their model, CEO compensation is paid as an immediate payment only when W reaches a threshold level.

range with a positive payment cannot be derived by the model of DeMarzo and Sannikov (2006). The reason is that in their model, dividends are not paid until the line of credit is fully repaid, and all excess cash flows are paid as dividends once the line of credit is fully repaid.

The existing theories for why firms smooth their dividends are based on asymmetric information, agency considerations, external finance costs, or tax planning (for a review of the literature, see Leary and Michaely (2011)). However, the empirical evidence is inconsistent with the asymmetric information model, although more consistent with the agency conflict model (see Leary and Michaely (2011)). A recent paper by Baker and Wurgler (2010) indicates that firms will smooth their dividends more the more that loss-averse investors value dividends relative to a reference point of prior dividends. Our explanation of dividend smoothing is similar to theirs in its use of the loss-aversion model, even though our framework is based on the continuous-time agency model.

6. Conclusion

In this paper, we explored a continuous-time agency model in which the agent has loss-aversion preferences, as introduced by prospect theory (Kahneman and Tversky (1979)). To formalize the model, we generalized the continuous-time agency model of DeMarzo and Sannikov (2006) by incorporating loss aversion.

Our main results are summarized as follows.

(i) The optimal contract includes a range of the agent's continuation payoff in which he is rewarded with fixed cash compensation. At each end of the flat segment, there is a range of the lower level of the agent's continuation payoff in which the fixed cash compensation is reduced according to his continuation payoff when his continuation payoff decreases, and a range of the higher level of the agent's continuation payoff in which all excess cash flows are paid immediately as the exercise of option grants. The sensitivity of the agent's compensation to his continuation payoff is larger in the latter nonflat range than in the former.

(ii) The introduction of loss aversion induces investors to reward the agent earlier.

(iii) An increase in the agent's degree of loss aversion induces investors to use stronger incentives.

(iv) As in DeMarzo and Sannikov (2006), the optimal contract is implemented by the combination of equity, long-term debt, and a line of credit.

(v) Our theoretical findings provided some explanations for the evolution of CEO compensation in the United States since World War II as well as the low level of stability of CEOs' fractional equity ownership in the same period, and for the corporate dividend-smoothing policy.

For the sake of the tractability of our dynamic contracting problem, we have assumed throughout this paper that reference income is exogenous, and that the agent is risk neutral. However, exogenous reference income may not always be plausible, although it provides a useful benchmark. For example, de Meza and Webb (2007) consider the median reference wage for the endogenous reference point. It would be interesting to use such an index as a reference point. It would also be interesting to examine the interaction between the degrees of loss aversion and risk aversion under the continuous-time agency framework. We could then address the problem of private savings, as highlighted in Edmans, Gabaix, Sadzik, and Sannikov (2012) and He (2012).

Appendix (For Online Publication except the proofs of Lemma 2, Proposition 1, and Lemma A1)

Proof of Lemma 1: Using procedures similar to those in the proof of Lemma 1 in the Appendix in DeMarzo and Sannikov (2006), we can prove this lemma. Note that the loss-aversion component does not modify the proof. ■

Proof of Lemma 2: We first show that the agent's continuation value W_t evolves according to (8). $W_t(\widehat{Y})$ is the agent's continuation payoff if $\widehat{Y}_s$, $0 \leq s \leq t$ were the true cash flows and the agent reported truthfully. Hence, without loss of generality, we can prove (8) when the agent truthfully reports $\widehat{Y} = Y$.[20] Now, let

$$V_t = \int_0^t e^{-\gamma s} \left\{ dI_s(Y) + \theta(dI_s(Y), a) \left[dI_s(Y) - ads \right] \right\} + e^{-\gamma t} W_t(Y). \qquad \text{(A1)}$$

We now prove that V_t is a martingale. Indeed, it follows from (7) and (A1) that for $s < t$,

$$\begin{aligned}
& E\left(V_t \mid \mathcal{F}_s\right) \\
& = E(V_t - V_s + V_s \mid \mathcal{F}_s) \\
& = E\left\{ \int_s^t e^{-\gamma k} \left[dI_k(Y) + \theta(dI_k(Y), a)(dI_k(Y) - adk) \right] + e^{-\gamma t} W_t(Y) \mid \mathcal{F}_s \right\} \\
& - e^{-\gamma s} W_s(Y) + V_s \\
& = E\left\{ \int_s^t e^{-\gamma k} \left[dI_k(Y) + \theta(dI_k(Y), a)(dI_k(Y) - adk) \right] \right. \\
& \left. + e^{-\gamma t} \left[\int_t^\tau e^{-\gamma (k-t)} [dI_k(Y) + \theta(dI_k(Y), a)(dI_k(Y) - adk)] + e^{-\gamma(\tau - t)} R \right] \mid \mathcal{F}_s \right\} \\
& - e^{-\gamma s} W_s(Y) + V_s \\
& = E\left\{ \int_s^\tau e^{-\gamma k} \left[dI_k(Y) + \theta(dI_k(Y), a)(dI_k(Y) - adk) \right] + e^{-\gamma \tau} R \mid \mathcal{F}_s \right\} \\
& - e^{-\gamma s} W_s(Y) + V_s
\end{aligned}$$

[20] By Lipschitz continuity of $Y_t - \widehat{Y}_t$, the probability measures over the paths of Y and $\widehat{Y}$ are equivalent.

$$= e^{-\gamma s} E\left\{\int_s^\tau e^{-\gamma(k-s)}\left[dI_k(Y) + \theta(dI_k(Y), a)(dI_k(Y) - adk)\right] + e^{-\gamma(\tau-s)} R \mid \mathcal{F}_s\right\}$$
$$- e^{-\gamma s} W_s(Y) + V_s$$
$$= e^{-\gamma s} W_s(Y) - e^{-\gamma s} W_s(Y) + V_s = V_s.$$

Because V_t is a martingale, it follows from the martingale representation theorem that there is a process $\beta \equiv \{\beta_t : 0 \le t \le \tau\}$ such that

$$dV_t = e^{-\gamma t}\beta_t(Y)(dY_t - \mu dt),$$

where $dY_t - \mu dt$ is a multiple of the standard Brownian motion. Differentiating (A1) with respect to t, we have

$$\begin{aligned} dV_t &= e^{-\gamma t}\beta_t(Y)(dY_t - \mu dt) \\ &= e^{-\gamma t}\left[dI_t(Y) + \theta(dI_t(Y), a)(dI_t(Y) - adt)\right] - \gamma e^{-\gamma t} W_t(Y) dt + e^{-\gamma t} dW_t(Y). \end{aligned}$$

Thus, given $dI_t = dC_t$ when $\widehat{Y} = Y$, (8) is obtained.

We next prove that (9) is the incentive-compatible constraint. If the agent diverts $dY_t - d\widehat{Y}_t$ at time t, he gains immediate income $\lambda(dY_t - d\widehat{Y}_t)$ but loses $\beta_t(dY_t - d\widehat{Y}_t)$ in future expected payoffs. In addition, he reduces disutility from loss aversion, $\lambda\theta(dC_t(Y), a)(dY_t - d\widehat{Y}_t)$. Hence, reporting strategy $\widehat{Y}$ gives the agent the payoff

$$W_0 + E\left\{\int_0^\tau e^{-\gamma t}\left[\lambda(dY_t - d\widehat{Y}_t) + \lambda\theta(dC_t(Y), a)(dY_t - d\widehat{Y}_t) - \beta_t(dY_t - d\widehat{Y}_t)\right]\right\}, \tag{A2}$$

where W_0 denotes the agent's payoff under truth telling. We see that if $\beta_t \ge \lambda[1 + \theta(dC_t, a)]$ for all t, (A2) is maximized when the agent chooses $dY_t = d\widehat{Y}_t$ for any t because the agent cannot overreport cash flows. If $\beta_t < \lambda[1 + \theta(dC_t, a)]$ on a set of positive measures, the agent is better off underreporting on this set than always telling the truth. ■

Proof of Proposition 1: (i) Because $\widetilde{W} + \theta a < W^1$, we divide the range of W into the following four segments: (a) $W \le \widetilde{W}$, (b) $\widetilde{W} < W \le \widetilde{W} + (1 + \theta)a$, (c) $\widetilde{W} + (1 + \theta)a < W \le W^1 + a$, and (d) $W^1 + a < W$. It follows from the definition of $\widetilde{W}$ that if $dI \le a$, the optimal contract does not pay out any cash ($dI = 0$) until W_t exceeds the reflecting barrier $\widetilde{W}$. Hence, when $W \le \widetilde{W}$, it is optimal for investors to set $dI = 0$. When $\widetilde{W} < W \le \widetilde{W} + (1 + \theta)a$, it follows from $b'(W) < 0$ that investors find it optimal to achieve $b'(\widetilde{W}) = -\frac{1}{1+\theta}$ by paying $dI = \frac{W-\widetilde{W}}{1+\theta}$ and returning W to $\widetilde{W}$.[21] Note that for this range of W, the condition of $dI \le a$ is satisfied. Thus, when $\widetilde{W} < W \le \widetilde{W} + (1 + \theta)a$, we have $dI = \frac{W-\widetilde{W}}{1+\theta}$. When $\widetilde{W} + (1 + \theta)a < W \le W^1 + a$, it would still be optimal for investors to achieve $b'(\widetilde{W}) = -\frac{1}{1+\theta}$ by paying $dI = \frac{W-\widetilde{W}}{1+\theta}$ and returning W to $\widetilde{W}$ if $dI \le a$ were satisfied. However, in this range of W, we see that $dI > a$ would hold if the agent were paid $dI = \frac{W-\widetilde{W}}{1+\theta}$. As a result, it is optimal for investors to increase dI up to a, but not beyond a. Finally, when $W^1 + a < W$, it follows from $b'(W) < 0$ and $W - W^1 > a$ that it is optimal for investors to achieve $b'(W^1) = -1$ by paying $dI = W - W^1$ and returning W to W^1. Note that the optimal contract pays out cash $dI > 0$ for $W \in (W^1 + a, \infty)$ because W_t exceeds the reflecting barrier W^1 when $dI > a$.

(ii) In this case, because $W^1 + a \le \widetilde{W} + (1 + \theta)a$, we divide the range of W into the following three segments: (a) $W \le \widetilde{W}$, (b) $\widetilde{W} < W \le W^1 + a$, and (c) $W^1 + a < W$. When $\widetilde{W} < W \le W^1 + a$, investors can achieve $b'(\widetilde{W}) = -\frac{1}{1+\theta}$ by paying $dI = \frac{W-\widetilde{W}}{1+\theta}$ and returning W to $\widetilde{W}$. The reason is that this payment policy ensures that $dI \le a$ holds under the assumption of $W^1 \le \widetilde{W} + \theta a$. Hence, it is optimal for investors to set $dI_t = 0$ for $W \le \widetilde{W}$, and $dI_t = \frac{W-\widetilde{W}}{1+\theta}$ for $\widetilde{W} < W \le W^1 + a$. In fact, if the agent is paid $dI = W - W^1$, $dI > a$ holds as long as $W^1 + a < W$. Thus, when $W^1 + a < W$, it follows from $b'(W) < 0$ and $W - W^1 > a$ that it is optimal for investors to achieve $b'(W^1) = -1$ by paying $dI = W - W^1$ and returning W to W^1. ■

[21] Because $b'(\widetilde{W}) = -\frac{1}{1+\theta}$ and $b''(W) < 0$, as verified in Proposition 2, we see that $b'(W) < 0$ for any $W \ge \widetilde{W}$.

Proof of Proposition 2: We start by showing that $b_-(W)$ (or $b_+(W)$) is strictly concave on $[R, \widetilde{W})$ (or $W \in (\widetilde{W} + (1+\theta)a, W^1 + a)$).

Lemma A1. *The function $b_-(W)$ is strictly concave on $[R, \widetilde{W})$.*

Proof: Define the function $F_-(W)$ as $F_-(W) = W + (1+\theta)b_-(W)$. Then, using (14a), $F_-(W)$ satisfies the following differential equation:

$$
\begin{aligned}
rF_-(W) = {} & -(\gamma - r)W + (1+\theta)\mu - \theta a + (\gamma W + \theta a)\, F_-'(W) \\
& + \frac{1}{2}\lambda^2(1+\theta)^2\sigma^2 F_-''(W), \qquad \text{for } W \in [R, \widetilde{W}], \qquad \text{(A3)}
\end{aligned}
$$

with the boundary conditions

$$
F_-(R) = R + (1+\theta)L, \;\; F_-'(\widetilde{W}) = 0, \;\; \text{and} \;\; F_-''(\widetilde{W}) = 0.
$$

Let us now focus on W in the neighborhood of the reflection barrier of $W = \widetilde{W}$, that is, $W \in [\widetilde{W} - \epsilon_1, \widetilde{W}]$. Differentiating (A3) with respect to W and using the boundary conditions yields

$$
\frac{dF_-''(W)}{dW} \simeq \frac{2(\gamma - r)}{\lambda^2(1+\theta)^2\sigma^2} > 0, \qquad \text{for sufficiently small } \epsilon_1 > 0.
$$

Because $F_-''(\widetilde{W}) = 0$ and $\frac{dF_-''(W)}{dW} > 0$, this implies that there exists $\epsilon_2 > 0$ such that $F_-''(W) < 0$ over the interval $[\widetilde{W} - \epsilon_2, \widetilde{W})$. In addition, as $F_-'(\widetilde{W}) = 0$ and $F_-''(W) < 0$ over the interval $[\widetilde{W} - \epsilon_2, \widetilde{W})$, we also have $F_-'(W) > 0$ over the interval $[\widetilde{W} - \epsilon_2, \widetilde{W})$.

Now, it follows from (A3) that

$$
\begin{aligned}
F_-''(W) &= \frac{rF_-(W) + (\gamma - r)W - (1+\theta)\mu + \theta a - (\gamma W + \theta a)\, F_-'(W)}{\frac{1}{2}\lambda^2(1+\theta)^2\sigma^2} \\
&= \frac{G_-(W) - (\gamma W + \theta a)\, F_-'(W)}{\frac{1}{2}\lambda^2(1+\theta)^2\sigma^2}, \qquad \text{(A4)}
\end{aligned}
$$

where $G_-(W) \equiv rF_-(W) + (\gamma - r)W - (1+\theta)\mu + \theta a$. It also follows from

(A3) with the boundary conditions that

$$G_{-}(\widetilde{W}) = 0. \tag{A5}$$

Furthermore,

$$G'_{-}(W) = rF'_{-}(W) + \gamma - r. \tag{A6}$$

Then, using (A4)–(A6), we can show that $F''_{-}(W) < 0$ for any $W \in [R, \widetilde{W})$ if $F'_{-}(W) > 0$ for any $W \in [R, \widetilde{W})$.[22] To prove $F'_{-}(W) > 0$ for any $W \in [R, \widetilde{W})$, suppose that $F'_{-}(W) \leq 0$ for some $W \in [R, \widetilde{W} - \epsilon_2)$. Let $W^\circ \equiv \sup\{W \leq \widetilde{W} - \epsilon_2 : F'_{-}(W) \leq 0\}$. Because we have already shown that $F'_{-}(W) > 0$ for all $W \in [\widetilde{W} - \epsilon_2, \widetilde{W})$, we must have $F'_{-}(W^\circ) = 0$ and $F'_{-}(W) > 0$ for all $W \in (W^\circ, \widetilde{W})$. However, given (A5) and (A6), this implies that $G_{-}(W) < 0$ for all $W \in (W^\circ, \widetilde{W})$. Thus, we must have $F''_{-}(W) < 0$ for all $W \in (W^\circ, \widetilde{W})$. On the other hand, $F'_{-}(\widetilde{W}) = F'_{-}(W^\circ) + \int_{W^\circ}^{\widetilde{W}} F''_{-}(W)dW$. It follows from $F'_{-}(\widetilde{W}) = 0$ that

$$F'_{-}(W^\circ) = -\int_{W^\circ}^{\widetilde{W}} F''_{-}(W)dW. \tag{A7}$$

As $F''_{-}(W) < 0$ for all $W \in (W^\circ, \widetilde{W})$, the right-hand side of (A7) is positive. This implies that $F'_{-}(W^\circ) > 0$, which is a contradiction. Thus, we obtain $F'_{-}(W) > 0$ for all $W \in [R, \widetilde{W})$. As a result, we must also have $F''_{-}(W) < 0$ for all $W \in [R, \widetilde{W})$. Because $b''_{-}(W) = F''_{-}(W)$, we complete the proof of this lemma. ‖

Lemma A2. The function $b_{+}(W)$ is strictly concave on $W \in (\widetilde{W} + (1 + \theta)a, W^1 + a)$.

Proof: Define the function $F_{+}(W)$ as $F_{+}(W) = W + b_{+}(W)$. Using (14b),

[22] Given (A5) and (A6), note that $G_{-}(W) < 0$ for any $W \in [R, \widetilde{W})$ if $F'_{-}(W) > 0$ for any $W \in [R, \widetilde{W})$.

$F_+(W)$ satisfies the following differential equation:

$$rF_+(W) = -(\gamma - r)W + \mu + (\gamma W - a)\, F'_+(W) + \frac{1}{2}\lambda^2(1+\theta)^2\sigma^2 F''_+(W),$$
$$\text{for } W \in (\widetilde{W} + (1+\theta)a,\ W^1 + a], \tag{A8}$$

with the boundary conditions

$$F_-(\widetilde{W}+(1+\theta)a) = F_+(\widetilde{W}+(1+\theta)a),\ F'_+(W^1+a) = 0,\ \text{and } F''_+(W^1+a) = 0.$$

Then, repeating a procedure similar to that of Lemma A1, we can prove this lemma. ‖

Now, note that

$$b'_-(W) = -\frac{1}{1+\theta}, \qquad \text{for all } W \in [\widetilde{W},\ \widetilde{W} + (1+\theta)a],$$
$$b'_+(W) = -1, \qquad \text{for all } W \in [W^1 + a,\ \infty).$$

In addition, the optimal payment conditions imply that $b'_-(W) \geq -\frac{1}{1+\theta}$ for all $W \in [R, \widetilde{W})$ and $b'_+(W) \geq -1$ for all $W \in (\widetilde{W} + (1+\theta)a, W^1 + a]$. In the neighborhood of $W = \widetilde{W} + (1+\theta)a$, investors pay $dI = \frac{W-\widetilde{W}}{1+\theta}$ for $W \leq \widetilde{W} + (1+\theta)a$, and $dI = a$ for $W > \widetilde{W} + (1+\theta)a$. Because $\lim_{W\to\widetilde{W}+(1+\theta)a-0} dI = \lim_{W\to\widetilde{W}+(1+\theta)a+0} dI = a$ and the cash payment satisfies $dI \leq a$ at $W = \widetilde{W} + (1+\theta)a$, we also have $\lim_{W\to\widetilde{W}+(1+\theta)a-0} b'_-(W) = \lim_{W\to\widetilde{W}+(1+\theta)a+0} b'_+(W) = -\frac{1}{1+\theta}$. Thus, combining the results of Lemmas A1 and A2 with the above findings, we verify that the function $b(W)$ is concave on $[R, \infty)$.

Next, for any incentive-compatible contract (τ, I), define

$$J_t = \int_0^t e^{-rs}(dY_s - dI_s) + e^{-rt}b(W_t),$$

where W_t evolves according to (8) with $dC_t = dI_t$. Note that the process J is such that J_t is $\mathcal{F}_t$-measurable. It follows from Ito's lemma that under an arbitrary incentive-compatible contract (τ, I),

$$
\begin{aligned}
e^{rt}dJ_t = & \left[\mu + \gamma W_t b'(W_t) + \frac{1}{2}\beta_t^2\sigma^2 b''(W_t) - rb(W_t)\right] dt - [1 + b'(W_t)]dI_t \\
& - \theta(dI_t, a)(dI_t - adt)b'(W_t) + [1 + \beta_t b'(W_t)]\sigma dZ_t. \qquad \text{(A9)}
\end{aligned}
$$

Thus, for $W_t \in [R, \widetilde{W}]$, it is found from (14a) and (A9) that

$$
\begin{aligned}
e^{rt}dJ_t = & \frac{1}{2}\left[\beta_t^2 - \lambda^2(1+\theta)^2\right]\sigma^2 b''_{-}(W_t)dt - \left\{1 + [1 + \theta(dI_t, a)]b'_{-}(W_t)\right\} dI_t \\
& + [\theta(dI_t, a) - \theta]\, b'_{-}(W_t)adt + [1 + \beta_t b'_{-}(W_t)]\sigma dZ_t. \qquad \text{(A10)}
\end{aligned}
$$

The first component of the right-hand side of (A10) is less than or equal to 0 because of $b''_{-}(W_t) < 0$ for $W_t \in [R, \widetilde{W})$ from Lemma A1 and $b''_{-}(\widetilde{W}) = 0$, and $\beta_t \geq \lambda(1 + \theta)$ for $W_t \in [R, \widetilde{W}]$ from Lemma 2. The sum of the second and third components is also less than or equal to zero. The reason is as follows. If $dI_t \leq adt$, then $\theta(dI_t, a) = \theta$. Given that $b'_{-}(W) \geq -\frac{1}{1+\theta}$ for $W_t \in [R, \widetilde{W}]$ and $dI_t \geq 0$, the sum of these components is less than or equal to zero. If $dI_t > adt$, then $\theta(dI_t, a) = 0$. Thus, the sum of these components equals $-dI_t - [dI_t + \theta adt]b'_{-}(W_t)$. If $b'_{-}(W_t) \geq 0$, this is less than or equal to zero. If $b'_{-}(W_t) < 0$, it follows from $dI_t > adt$ and $b'_{-}(W_t) \geq -\frac{1}{1+\theta}$ that the sum of these components is less than or equal to zero.

For $W_t \in (\widetilde{W} + (1 + \theta)a,\ W^1 + a]$, it follows from (14b) and (A9) that

$$
\begin{aligned}
e^{rt}dJ_t = & \frac{1}{2}\left[\beta_t^2 - \lambda^2(1+\theta)^2\right]\sigma^2 b''_{+}(W_t)dt + \left\{1 + [1 + \theta(dI_t, a)]b'_{+}(W_t)\right\}(adt - dI_t) \\
& + [1 + \beta_t b'_{+}(W_t)]\sigma dZ_t. \qquad \text{(A11)}
\end{aligned}
$$

Again, given Lemma A2, $b''_{+}(W^1 + a) = 0$, and Lemma 2, the first component of the right-hand side of (A11) is less than or equal to zero when $W_t \in (\widetilde{W} + (1 + \theta)a,\ W^1 + a]$. The second component is also less than or equal to zero. The reason is as follows. If $dI_t \leq adt$, then $\theta(dI_t, a) = \theta$. Given $\lim_{W \to \widetilde{W}+(1+\theta)a+0} b'_{+}(W) = -\frac{1}{1+\theta}$, $b'_{+}(W^1 + a) = -1$, and Lemma A2, we have $-\frac{1}{1+\theta} > b'_{+}(W) > -1$ for

$W_t \in (\widetilde{W} + (1 + \theta)a,\, W^1 + a]$. Thus, this component is less than or equal to zero. If $dI_t > adt$, then $\theta(dI_t, a) = 0$. Hence, it follows from $b'_+(W) \geq -1$ that this component is also less than or equal to zero.

For $W_t \in (\widetilde{W},\, \widetilde{W} + (1 + \theta)a]$, under an arbitrary incentive-compatible contract (τ, I), it follows from (15a), (16), and (A9) that

$$\begin{aligned} e^{rt}dJ_t &= \left[\mu - \frac{\gamma W_t}{1+\theta} - rb_-(W_t)\right]dt - \frac{\theta}{1+\theta}dI_t + \frac{\theta(dI_t, a)(dI_t - adt)}{1+\theta} \\ &\quad + \left(1 - \frac{\beta_t}{1+\theta}\right)\sigma dZ_t \\ &= -\frac{(\gamma - r)(W_t - \widetilde{W})}{1+\theta}dt + \frac{\left[\theta(dI_t, a) - \theta\right](dI_t - adt)}{1+\theta} + \left(1 - \frac{\beta_t}{1+\theta}\right)\sigma dZ_t. \end{aligned} \tag{A12}$$

As $W_t \in (\widetilde{W},\, \widetilde{W} + (1 + \theta)a]$, the first component of the right-hand side of (A12) is less than zero. The second component is also less than or equal to zero because $\theta(dI_t, a) = \theta$ for $dI_t \leq adt$ and $\theta(dI_t, a) = 0$ for $dI_t > adt$.

Equations (A10)–(A12) imply that the process J is an $\mathcal{F}_t$-supermartingale up to time $t = \tau$. Furthermore, the process J will be an $\mathcal{F}_t$-martingale for $W_t \in [R, \widetilde{W}]$ and $W_t \in (\widetilde{W} + (1 + \theta)a,\, W^1 + a]$ under the contract satisfying the conditions of this proposition because $\beta_t = \lambda(1 + \theta)$, $dI_t = 0$, and $\theta(dI_t, a)$ $= \theta$ for $W_t \in [R, \widetilde{W}]$, and $\beta_t = \lambda(1 + \theta)$ and $dI_t = adt$ for $W_t \in (\widetilde{W} + (1 + \theta)a,$ $W^1 + a]$. The process J will also be an $\mathcal{F}_t$-martingale for $W_t \in (\widetilde{W}, \widetilde{W} + (1 +$ $\theta)a]$ because $\theta(dI_t, a) = \theta$ and the agent returns W_t to $\widetilde{W}$ immediately. Thus, the process J is an $\mathcal{F}_t$-martingale under the contract satisfying the conditions of this proposition up to time $t = \tau$.

We now evaluate the principal's expected payoff for an arbitrary incentive-compatible contract (τ, I), which equals

$$E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right].$$

Using the definition of process J, we show that under any arbitrary incentive-compatible contract (τ, I) and any $t \in [0, \infty)$,

$$\begin{aligned}
&E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right] \\
&= E(J_{t\wedge\tau}) + E\left\{\mathbf{1}_{t\leq\tau}\left[\int_t^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L - e^{-rt}b(W_t)\right]\right\} \\
&\leq b(W_0) + E\left\{\mathbf{1}_{t\leq\tau}\left[\int_t^\tau e^{r(t-s)}(dY_s - dI_s) + e^{r(t-\tau)}L - b(W_t) \mid \mathcal{F}_t\right]\right\} e^{-rt},
\end{aligned} \tag{A13}$$

where the inequality follows from the fact that $J_{t\wedge\tau}$ is a supermartingale and $J_0 = b(W_0)$. In addition,

$$E\left\{\mathbf{1}_{t\leq\tau}\left[\int_t^\tau e^{r(t-s)}(dY_s - dI_s) + e^{r(t-\tau)}L \mid \mathcal{F}_t\right]\right\} < \frac{\mu}{r} - W_t. \tag{A14}$$

This is because the right-hand side of (A14) is the upper bound on the principal's expected profit under the first-best contract. Combining (A13) and (A14), we obtain

$$E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right] \leq b(W_0) + E\left\{\mathbf{1}_{t\leq\tau}\left[\frac{\mu}{r} - W_t - b(W_t)\right]\right\} e^{-rt}. \tag{A15}$$

Using $b'(W_t) \geq -1$ and $b(R) = L$, we have $W_t + b(W_t) \geq L$ for any $W_t \geq R$. Hence, applying this to (A15), we have

$$E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right] \leq b(W_0) + e^{-rt}E\left[\mathbf{1}_{t\leq\tau}\left(\frac{\mu}{r} - L\right)\right].$$

Taking $t \to \infty$ yields

$$E\left[\int_0^\tau e^{-rs}(dY_s - dI_s) + e^{-r\tau}L\right] \leq b(W_0).$$

Let (τ^*, I^*) be a contract that satisfies the conditions of the proposition. This contract is incentive compatible because $\beta_t = \lambda(1+\theta)$ when $dI_t \leq a$. Furthermore, under this contract, the process J is a martingale until time τ because

$b'(W_t)$ stays bounded. Therefore, the payoff $b(W_0)$ is achieved with equality under (τ^*, I^*). ■

Proof of Proposition 3: As $\rho \leq r$, we focus on the case of $\rho = r$ without loss of generality because maintaining savings is most attractive in this case. We also generalize the setting by allowing the agent to save not only in his own account but also within the firm. Let S_t^f denote the savings within the firm, which evolves according to

$$dS_t^f = rS_t^f dt + dY_t - d\widehat{Y}_t - dQ_t,$$

where dQ_t is the agent's diversion of the cash flows to his own account. The agent's balance S_t also evolves by

$$dS_t = rS_t dt + [dQ_t]^\lambda + dI_t - dC_t.$$

Note that the agent bears the cost of diversion when he diverts the cash flows to his own account. For an arbitrary feasible strategy $(C, \widehat{Y})$ of the agent, let

$$\widehat{V}_t = \int_0^t e^{-\gamma s} \left[dC_s + \theta(C_s, a)(dC_s - ads)\right] + e^{-\gamma t}(S_t + \lambda S_t^f + W_t).$$

To prove that $\widehat{V}_t$ is a supermartingale, let us note that

$$e^{\gamma t} d\widehat{V}_t = dC_t + \theta(dC_t, a)(dC_t - adt) + dS_t - \gamma S_t dt + \lambda(dS_t^f - \gamma S_t^f dt) + dW_t - \gamma W_t dt.$$

It follows from (19) and the definitions of dS_t and dS_t^f that

$$\begin{aligned} e^{\gamma t} d\widehat{V}_t = &-(\gamma - r)(S_t + \lambda S_t^f)dt + [dQ_t]^\lambda - \lambda dQ_t \\ &+ \lambda[1 + \theta(dC_t, a)]\sigma dZ_t + \lambda\theta(dC_t, a)(d\widehat{Y}_t - dY_t). \end{aligned}$$

$\widehat{V}_t$ is now a supermartingale until time τ because $\gamma > r$, $[dQ_t]^\lambda - \lambda dQ_t$ is nonincreasing, the savings balances are nonnegative, overreporting by putting the agent's own money back into the project is not allowed (that is, $d\widehat{Y}_t \leq dY_t$), and W_t is bounded from below (that is, $W_t \geq R$). Furthermore, $\widehat{V}_t$ is a martingale if W_t is bounded from above, if there are no savings (that is, $S_t = S_t^f = 0$), and if the agent reports truthfully ($d\widehat{Y}_t = dY_t$ and $dQ_t = 0$). Hence,

$$\begin{aligned} W_0 &= \widehat{V}_0 \geq E(\widehat{V}_\tau) \\ &= E\left[\int_0^\tau \left[e^{-\gamma s} dC_s + \theta(C_s, a)(dC_s - ads)\right] + e^{-\gamma\tau}(S_\tau + \lambda S_\tau^f + R)\right], \end{aligned}$$

with equality if the agent maintains zero savings and reports truthfully. This statement holds even if $Y_t - \widehat{Y}_t$ is not Lipschitz continuous. ■

Proof of Proposition 4: For $dDiv_t$ given by (22), the credit line balance, M_t, evolves according to

$$dM_t = \gamma M_t + x_t dt + dDiv_t - d\widehat{Y}_t, \tag{A17}$$

where we can assume that $d\widehat{Y}_t$ is such that $dM_t \geq 0$. It follows from (4), (20b), (20c), (21), Proposition 1(i), and the definitions of $M_{\widetilde{W}}$ *and* $M_{\widetilde{W}+(1+\theta)a}$ that

$$\begin{aligned} \lambda(1+\theta)x_t dt &= \lambda(1+\theta)rD_t dt \\ &= \left[\lambda(1+\theta)\mu - \gamma R - \gamma\lambda(1+\theta)C^L\right] dt - \theta(dI_t, a)(-dI_t + adt). \end{aligned} \tag{A18}$$

It also follows from (21) with (A17) and (A18) that

$$\begin{aligned} dW_t &= -\lambda(1+\theta)dM_t = -\lambda(1+\theta)\left(\gamma M_t dt + x_t dt + dDiv_t - d\widehat{Y}_t\right) \\ &= \gamma W_t dt + \theta(dI_t, a)(-dI_t + adt) - \lambda(1+\theta)dDiv_t + \lambda(1+\theta)(d\widehat{Y}_t - \mu dt). \end{aligned} \tag{A19}$$

Let $dC_t = dI_t = \lambda(1+\theta)dDiv_t$, and note that $\theta(dI_t, a) = \theta$ because $\lambda(1+\theta)dDiv_t \leq a$ for $M_t > 0$. Then, it follows from Propositions 2 and 3 that the capital structure given by this proposition is optimal for the agent.

Under the capital structure proposed by this proposition and the agent's optimal strategy $(I, \widehat{Y}) = (C, \widehat{Y}) = (C^*, Y)$, the principal's expected utility equals

$$E\left[\int_0^{\tau(C^*,Y)} e^{-rs}\left(dY_s - dC_s^*\right) + e^{-r\tau(C^*,Y)}L \mid \mathcal{F}_0\right] - K,$$

where $\tau(C^*, Y) = \inf\{t \mid W_t = R\} = \tau^*(Y)$. Note that the agent's continuation utility W_t evolves according to (A19) (that is, (19)), as in the optimal contract. In addition, it follows from (20c) and (21) that $M_t = M_{\widetilde{W}}$, $M_t = M_{\widetilde{W}+(1+\theta)a}$, and $M_t = 0$ implies $W_t = \widetilde{W}$, $W_t = \widetilde{W} + (1+\theta)a$, and $W_t = W^1 + a$, respectively. Hence, the capital structure given by this proposition is also optimal for the principal. We therefore conclude that the proposed capital structure implements the optimal contract. ■

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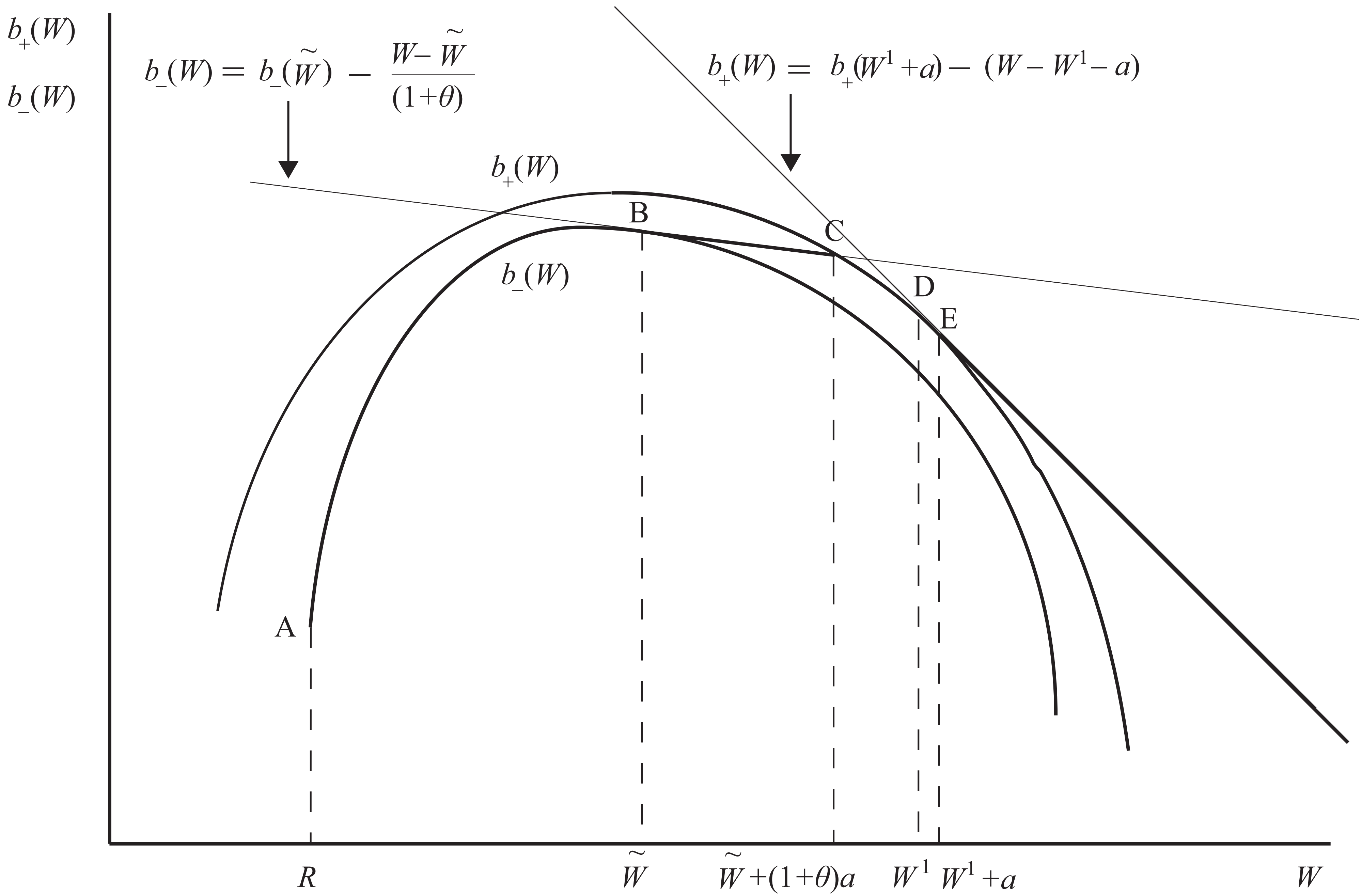

Figure 1. The investors' value function b(W)