Psychophysics of waiting time in decision under risk

Han R and Takahashi T Department of Behavioral Science, Center for Research in Social Science, Hokkaido University

1.Introduction

Life involves decision-makings over time and under uncertainty. Whether these two decisions have a common mechanism has been the area of controversy. In economics, the behavioral tendency associated with these decisions has been parameterized by time and risk attitudes independently. In contrast, behavioral psychologists regard decision under risk, what they call probability discounting, as having the same psychological mechanism with time discounting (i.e. intertemporal choice) (Rachlin et al, 1991). Moreover, the increasing empirical evidence has been accumulated to suggest systematic deviation from the normative assumption such as dynamic consistency of intertemporal choice (Samuelson, 1937) and the independent axiom of decision under risk (von Neumann and Morgenstern, 1947), To account for anomalies in intertemporal choice, Takahashi (2005) has proposed nonlinear psychophysical time. However, it is still unknown whether we can use the psychophysical time account for intertemporal chioce to explain the anomalies in decision under risk (probabilistic choice). Regarding this issue, we proposed that decision over time and under risk can both be explained by psychophysics of time (Takahashi, Han, Nakamura, 2012). In this paper, we present the empirical evidence that psychophysical time for delayed outcome and uncertain outcome (in virtually-repeated gambles) may account for anomalies in both intertemporal choice and probabilistic choice.

2. The anomaly in intertemporal choice and psychophysical time

Dynamic inconsistency of intertemporal choice is one of the major anomalies which drew much attention. People tend to discount the subjective value of delayed outcome, i.e. time discounting, of which the discount rate is assumed to be constant (i.e., time-consistent) in the economics (i.e. exponential discounting: $V(t) = V(0)\exp(-kt)$, V(t): time-discouted value of reward obtained at time t; k: time-discount rates)(Samuelson, 1937). Nevertheless, it has been empirically established that humans and animals show decreasing time discount rate, which is better described by a hyperbolic function ($V(t) = V(0)/(1+kt)$)(Ainslie, 1975; Green & Myerson, 2004). Such discounting behavior leads to time preference reversal (Strotz, 1955), which violates the assumption of rationality in neoclassical economic theory.

In order to solve dynamic inconsistency problem, Takahashi (2005) has introduced nonlinear psychophysical time into the time discount function. Based on Weber-Fechner law of psychophysics, the physical stimuli was psychologically represented in a logarithmic form. Hence, psychophysical time is:

$$\tau(t) = \alpha \ln(1+\beta t) \qquad \text{(Equation 1)}$$

where α and β are free parameters. We have thus time discounting with psychophysical time as follows:

$$V(t) = V(0) \cdot D(\tau(t)) = \frac{V(0)}{(1+\beta t)^{k\alpha}} \qquad \text{(Equation 2)}$$

where k is the degree of discounting at t=0, V(0)is the subjective value at t=0. Therefore, by introducing logarithmic psychophysical time, the time discount function in Equation 2 can express the hyperbola-like function. Han & Takahashi (2012) empirically found that the functional form of time discounting is closer to exponential than hyperbolic when logarithmic psychophysical time was introduced.

Furthermore, based on Tsallis' non-extensive thermostatistics, the q-exponential discount function is proposed to generalize the functional form of intertemporal choice (Cajuero, 2006;Takahashi, 2007). The q-exponential function is

$$V(t) = \frac{V(0)}{\left[1 + k(1-q)t\right]^{1/(1-q)}} \qquad \text{(Equation 3)}$$

where q is a free parameter and 1-q indicates the degree of deviation from the normative model (i.e. exponential model), can express both an exponential function (q→1) and a hyperbolic function (q=0) (Takahashi, 2007).

3. Nonlinear probability weighting function and psychophysical time

The decision makings of probabilistic outcome (i.e., decision under risk) require people to judge the benefit (utility) of each outcome as well as the likelihood of its occurrence. In von Neumann and Morgenstern's expected utility theory, individuals are assumed to decide by multiplying the objective probability and valuation (utility) of possible outcomes (von Neumann and Morgenstern, 1947). However, compelling experimental data have shown that people violated such assumption of rationality (i.e., the independence axiom). In other words, people decide as if they weight the utility of each outcome by its subjective probability, causing overestimation of low probability and underestimation of high probability (Kahneman & Tversky,1979; Prelec, 1998).

In behavioral psychology, devaluation (discounting) of uncertain outcome by people and animals is referred to as "probability discounting", where the degree of discounting represents risk attitude (i.e. risk aversion). The probability discounting was found to be in a hyperbolic form in terms of "odds-against" which corresponds to waiting time in repeated gambles (Rachlin et al, 1991; Green & Myerson, 2004). The hyperbolic probability discounting indicates that people over-devaluate likely (i.e., more immediate in repeated gambles) outcomes and under-devaluate unlikely (i.e., more delayed in repeated gambles) ones, which is in line with probability weighting function in prospect theory. Therefore, the hyperbolic probability discounting also represents the anomaly in probabilistic choice, consistent with Allais' paradox claimed in behavioral economics (Allais, 1953; Kahneman and Tversky, 1979).

According to Rachlin and colleagues (1991), time discounting theory also accounts for people's tendency to gamble whereby the probability (*p*) of wining repeated gambles can be converted to odds against (*O*) =1/p-1, that is the average waiting time until a win. Hence the probability discounting function is

$$V(O) = V(0)/(1 + k_p O) \qquad \text{(Equation 4)}$$

where k is the degree of discounting when O=0. We can now consider possible common psychophysical effects across intertemporal and probabilistic choices. If time discounting and probability discounting share the common underlying mechanism of waiting time until the rewards, it is possible that anomalies in decision under risk may also be related to nonlinear psychophysical effect of subjective waiting time in (virtual) repeated gambles. Given that psychophysical time for delayed outcome is nonlinear, we may conjecture that psychological time for waiting uncertain outcome may also be in a logarithmic form:

$$\tau(O) = \alpha \ln(1 + \beta O) \qquad \text{(Equation 5)}$$

where α and β are free parameters. Then we have a probability discount function with logarithmic time for uncertain outcome as

$$V(O) = V(0) \cdot D(\tau(O)) = \frac{V(0)}{(1 + \beta O)^{k\alpha}} \qquad \text{(Equation 6)}$$

This formula can express hyperbola-like probability discount functions. Hence it was conjectured that by introducing nonlinear psychophysical time of uncertain rewards, we can also "normalize" (i.e., exponentialize) the functional form of probability discounting.

This study sets out to empirically examine whether nonlinearity in subjective time for delayed outcome and uncertain outcome can normalize functional form of time discounting and probability discounting. We hypothesize that 1) both time discounting and probability discounting are in non-exponential forms. 2) psychophysical time for real and virtual delays in waiting for delayed and uncertain rewards are both in nonlinear forms. 3) both time discounting and probability discounting are less deviated from the normative models (i.e. closer to exponential form) once psychophysical time is introduced.

4. Method:

Participants: Thirty-three students (M=24, Mean age=19.79, SD=2.7) from Hokkaido University participated in our study.

Procedure: We asked participants to perform paper-pencil tasks of time discounting and probability discounting (Takahashi, 2007). They were asked to indicate the choice between a hypothetical 100,000 yen reward available with seven delays(1 week, 2 weeks, 1month, 6 month, 1 year, 5 years, 25 years) or seven probabilities (95%, 90%, 70%, 50%, 30%, 10%, 5%) and a certain reward of variable amount (from 0 yen ~ 100,000 yen) available immediately. For time perception tasks, they were asked to draw line on a 180mm scale to indicate their length of psychological time for delayed reward (e.g. "For how long do you feel you should wait until you receive the 100,000 yen 1 year later.") and uncertain reward (e.g. "For how long do you feel you should wait until you win 100,000 yen with probability of 50% in repeated gambles"). Data were analyzed as in our previous studies (e.g. Takahashi et al., 2007).

We used AIC (Akaike Information Criterion) to compare the fitness of the model, where the smaller values indicate the better fitness.

5. Results:

First, our study confirmed that both time discounting and probability discounting with physical time and objective odds against (:=1/p -1, proportioal to waiting time in virtural repeated gambles) were both in non-exponential forms. We fitted exponential, hyperbolic and q-exponential function to both discounting behaviors. The result of model fitness indicates that q-exponential function with the parameters yielding smallest AIC value fitted the behavioral data best (for time discounting see Table 1. and Figure 1. A; probability discounting, Table 1 and Figure 2 A). Furthermore, for both discount functions, parameter q was closer to 0 than to 1 indicating both functional forms were hyperbolic, rather than exponential (see Table 1).

Moreover, we also found that both psychophysical time for delayed outcome and uncertain outcome were in a nonlinear form. Note again that waiting time in virtual repeated gambles in probability discounting is mentioned as "odds against" := 1/p -1 in behavioral psychology. We fitted linear ($V(t) = \alpha t$)and power function($V(t) = \alpha t^{\beta}$) in addition to the log function to compare the fitness of the model. The results showed that the log function with parameters yielding smallest AIC best fitted the data for both psychophysical time for delayed and uncertain rewards (for subjective time see Table 2 and Figure 1. B; for subjective odds against see Table 2 and Figure 2 B;).

Finally, we demonstrated that the functional form of time discounting and probability discounting were both closer to exponential function after we introduced psychophysical time and odds against for delayed and uncertain rewards respectively. The results of AIC again revealed that the q-exponential function fitted both types of discounting behaviors (see Table 3). In contrast to the discount functions when physical measurements were used, the parameter q were closer to 1 rather than 0 for time discounting with subjective time and probability discounting with subjective odds against(i.e.)(see Table 3 and Figure 1 C and Figure 2 C). This confirmed our hypothesis that nonlinear psychophysical time can normalize (i.e. “exponentialize”) both time discount and probability discount functions, indicating that anomalies in both intertemporal and probabilistic choices may result from nonlinearity of psychophysical time.

6. Discussion

The account for anomalies in the intertemporal and risk decisions has been ongoing area of controversy. The theories have been proposed so far (e.g. prospect theory and Loewenstein and Prelec' theory) focused on value-related factors such as the nonlinear value function. In stark contrast, current study has adopted the perspective of psychophysical law of time perception, irrespective of value function, to account for those anomalies. Based on the idea that probabilistic choice and intertemporal choice share the same psychophysical mechanism of waiting time until receipt of outcome, we have provided tentative evidence that nonlinear psychophysical waiting time may be associated with anomalies in both decision over time and under risk, supporting our theoretical claim (Takahashi, Han, Nakamura, 2012).

Table 1. Parameters and AIC for time discounting and probability discounting of physical time and probability.

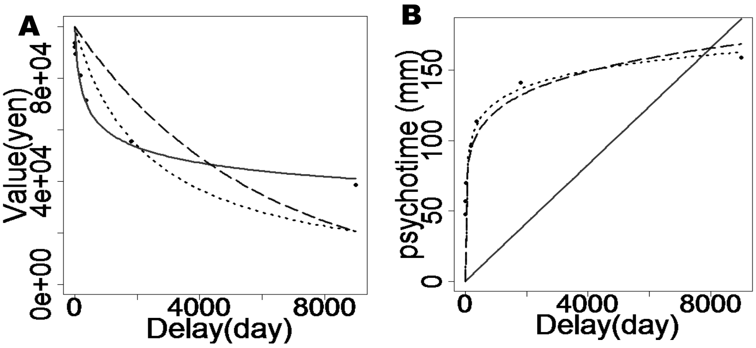
Model	Time discounting				Probability discounting			
	Exponential	Hyperbolic	q-exponential		Exponential	Hyperbolic	q-exponential	
	k	k	k	q	k	k	k	q
Parameter	0.011	0.03	19.9	-3.5	3.6	9.1	717.3	-0.8
AIC	148	147	135		153	147	142	

Table 2. Parameters and AIC of psychophysical time for delayed rewards and uncertain rewards.

Model	Psychophysical time for delayed reward					Psychophysical time for uncertain reward				
	Linear	Power		Logarithmic		Linear	Power		Logarithmic	
	α	α	β	α	β	α	α	β	α	β
Parameter	0.02	58.9	0.18	23.9	$3.4*10^{13}$	10.1	66.3	0.34	52.9	7047.4
AIC	84.4	63.4		59.4		75.9	60.5		55.8	

Table 3. Parameters and AIC for time discounting and probability discounting with subjective time for delayed and uncertain rewards.

Model	Time discounting (subjective time)				Probability discounting (subjective time)			
	Exponential k	Hyperbolic k	q-exponential k	q	Exponential k	Hyperbolic k	q-exponential k	q
Parameter	0.0051	0.012	0.00027	28.4	0.12	0.21	0.074	1.01
AIC	153	155	140		155	156	150	



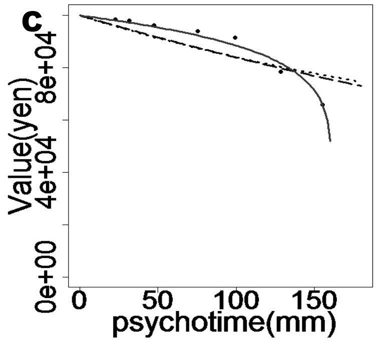
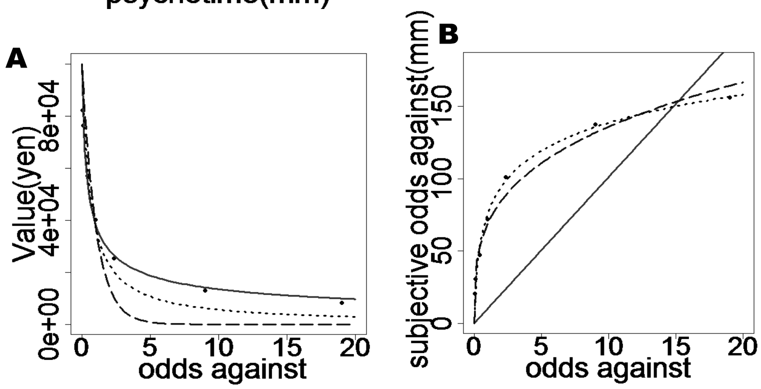


Figure 1. A) time discounting with physical time. B) psychophysical time with physical time. C) time discounting with psychophysical time. For graph A,C, the red, blue dash and black dot curves are q-exponential, hyperbolic and exponential function respectively. For graph B, the red, blue dash and black dot curves are linear, power and log function respectively



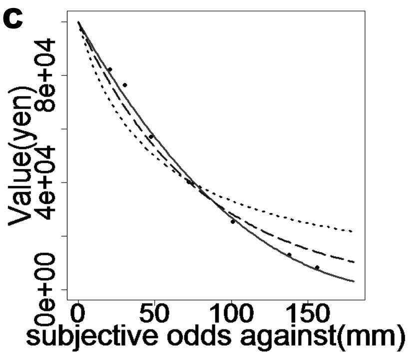


Figure 2. A) probability discounting with objective odds against (waiting time) B) psychophysical odds against with objective odds against C) probability discounting with psychophysical odds against. For graph A, C, the red, blue dash and black dot curves are q-exponential, hyperbolic and exponential function respectively. For graph B, the red, blue dash and black dot curves are linear, power and log function respectively

Reference

Ainslie, G. (1975). ***Psychol Bull*** 82, 463–496; Cajueiro (2006). ***Physica A*** 364 (2006) 385–388.; Green L, Myerson J.(2004). ***Psychol Bull.*** 130:769-792.;Takahashi, T. (2005). ***Medical Hypotheses***, 65, 691–693.; Rachlin H, Raineri A, Cross D(1991).***Jour Exp Anal Behv*** 55(2): 233-244; Samuelson, P. (1937). ***Rev. Econ. Stud***. 4, 155–161.;Strotz, R. H. (1955). ***Rev.Econ. Stud.***23, 165–180.; Takahashi, T, Oono, H., Radford M.H.B. (2007). ***Physica A*** 381 338-342.;Takahashi, Han &Nakamura (2012). ***Jour Behv Econ Fin***, (5)10-14.;Loewenstein, G. & Prelec, D.(1992).***Quart Jour Econ***, 107 (2), 573–597.; von Neumann, J., & Morgenstern, O. (1947).; Kahenman, D., Tversky, A. (1979). ***Econometrica***, 47: 263-291.