Temporal discounting and time perception of loss and gain

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Abstract:
We demonstrated the decreasing impatience is due to non-linearity in psychological time for both gain and loss. We examined how psychological time influence on temporal discounting of gain and loss in 50 college students. Psychological time in temporal discounting of gain and loss was nonlinear with physical time. Once we introduce psychological time, the discount function tends to be closer to an exponential model rather than a hyperbolic model.

1. Introduction

1.1. Intertemporal choice models

It has been well documented in the temporal discounting literatures that subjects devaluate the amount of rewards with time delay. The standard normative economic model assumes consistent intertemporal choice behavior, which follows the exponential equation (Samuelson, 1937):

$$V(D) = A\exp(-k_e D) \qquad \text{(Equation 1)},$$

where V is the subjective value of a reward, A is the amount of the reward, and D is the delay until the receipt of reward. A free parameter k_e indicates the degree to which a subject discounts the delayed reward, i.e. impatience. However, later evidence suggested that people show less rapid temporal discounting as delay increases, following hyperbolic equation (Rachlin, 1991;Ainslie, 1992;Mazur, 1978):

$$V=A/(1+ k_h D) \qquad \text{(Equation 2)},$$

where V, A, kh and D have the same definitions as Eq. (1). Unlike exponential discounting model, of which a discount rate = - (dV(D)/dD)/V= k_e is a constant value, a discount rate = -(dV(D)/dD)/V= $k_h/(1+ k_h D)$ is a decreasing function of delay D (decreasing impatience) in the hyperbolic discounting model, resulting in preference reversal over time thus inconsistency in intertemporal choice behavior. In addition to these models, Takahashi (2005) has proposed the *q*-exponential model of temporal discounting, which follows the equation:

$$V(D)=A/\exp_q(k_q D)=A/[1+(1-q)\, k_q D]1/(1-q) \qquad \text{(Equation 3)},$$

where expq(•) is the *q*-exponential function in the deformed algebra inspired by Tsallis' non-extensive thermodynamics, $q<1$ is a consistency parameter, (V, A, k_q and D have the same definitions as Eqs. (1) and (2)). It is to be noted that large *q* value corresponds to more consistent intertemporal choice; namely, $q\rightarrow 1$ corresponds to exponential discounting (complete consistency), while $q=0$, hyperbolic discounting (complete inconsistency).

1.2. Non-linear psychological time perception and temporal discounting

According to psychophysics, both the Stevens' power law and Weber–Fechner law of sensation suggest that people's internal representations of sensation for physical stimuli are in a power functional and a logarithmic functional form respectively, rather than in a linear form (Dehaene, 2003; Stevens, 1975). This indicates that psychological time is also in a non-linear form. Takahashi (2005) has theoretically demonstrated the mathematical equivalency between hyperbolic discount model with physical time and an exponential discount model with Weber-Fechner law of psychological time (when $\tau=a\ ln\ (1+bD\ )$(Equation 4), *a* and *b* are free parameters, then $V_{log}(D) =A exp(-k\tau)= A exp(-ka\ ln(1+bD))=A/(1+bD)^g$ (Equation 5), which is equivalent to Loewenstein & Prelec (1992)' generalized hyperbola. When $g=1$, Eq 5 is equivalent to simple hyperbolic function (Eq2). He also proposed that exponential discount model with subjective time-duration following

Steven's power law can also present decreasing impatience (when $\tau_s = aD^{s,}$ (Equation 6) then V_{power} $(D) = Aexp(-k\tau_s) = Aexp(-kD^s)$ (Equation 7)).

No study to date has systematically examined the effect of psychological time on temporal discounting of both gain and especially loss. Current study aims to experimentally examine 1) non-linearity of subjective time perception 2) the effect of psychological time on temporal discounting of gain and loss.

2. Method

2.1 Participants

Fifty (38 male and 12 female) Japanese college students were recruited to participate in the experiment. The average age was 19.9 (standard deviation= 0.9) years. Only four participants were smokers.

2.2 Procedures

Participants were seated individually and received a simple instruction that the monetary reward in this experiment was hypothetical, but required to think as though it were real money. They were asked to choose to receive or pay immediately (gain or loss) or with delay. The left column viewed from participants indicated the amounts of money that could be received (or pay) immediately (from 100,000 to 0 yen with each increment of 2,500 yen in ascending and descending order), and the right column indicated 100,000 yen that could be received (or pay) with a certain delay (1 week, 2 weeks, 1 month, 6 months, 1 year, 5 years and 25 years). Then they were asked to indicate the length of subjective time perception on an 18 cm line for all delays.

2.3 Data analysis

Indifference points of the intertemporal choice task were defined as the means of the largest adjusting amount in which the standard alternative choice and the smallest adjusting amount in which the adjusting alternative choice. The indifference points of individuals were calculated by averaging the indifference point in ascending and descending adjusting amounts of each individual.

We employed discounting models and psychophysical parameters introduced in the discounting equations above. For estimating the temporal discounting parameters (k_e, k_h, k_q and q), we fitted the three types of the discount models (the exponential discounting, hyperbolic discounting and q-exponential discounting, i.e. Eqs 1,2 and 3) to the average group data of indifference points. Further, we estimated time perception parameters (a, b and s) we fitted the three types of psychophysical models (Stevens' power law function, Weber-Fechner's logarithmic function and a linear model, Eqs 4 and 6). Lastly, in order to examine how non-linear time perception may affect the discount functional form, we again conducted model fitting of temporal discounting using psychological time perception rather than physical time. The fitness of each equation was estimated with AIC (Akaike Information Criterion) values, which smaller AIC values correspond to better fitting.

3. Results

3.1 Temporal discounting models with physical time

First, we fitted three types of temporal discounting models (exponential, hyperbolic and q-exponential model) to group average data of the indifference points at the delays (see Figure.1A for gain and Figure. 1B for loss).The results of model fitting showed that the orders of the AICs for group average data were [q-exponential discounting function < Hyperbolic discounting function < Exponential discounting function] for both gain and loss (see Table 1), which q-exponential discounting function fitted the behavioral data best.

3.2 Temporal discounting models with subjective time perception

Then we fitted three psychophysical models (Steven's power, Weber-Fechner logarithm and linear model) to subjective time perception. The results of model fitting showed that the orders of the AICs for group average data were [Weber-Fechner logarithmic function < Steven's power law function < Linear function] for both gain and loss (see Table 2), which Weber-Fechner logarithmic function fitted the behavioral data best. Both Weber-Fechner logarithmic function and Steven's power law function of psychological time indicate human have diminishing sensitivity to physical time. Furthermore, we fitted three types of temporal discounting models to group behavior data with Weber-Fechner logarithmic function of time perception (see Figure.1C for gain and Figure. 1D for loss). The results of model fitting showed that the orders of the AICs for group average data were [q-exponential discounting function < Exponential discounting function < Hyperbolic discounting function] for both gain and loss (see Table 2). Although the q-exponential model is still the best fitted model, the exponential model better fitted than hyperbolic model. Further, parameter q in the q-exponential model is greater than 1, indicating q is closer to exponential model than hyperbolic model (see Table 1), which also suggested the hyperbolicity has disappeared when using psychological time.

4. Discussion

Our study has experimentally demonstrated the effect of psychological time on temporal discounting of gain and loss. Preference reversal over time in hyperbolic temporal discounting has been a major topic of behavioral economics because the decreasing impatience means time inconsistent behavior, which violates the assumption of rationality in the standard economic model. Our present study indicates that people may use internally represented psychological time, which tend to underestimate the time length as delay increases, to make intertermporal choice. When we introduced psychological time in the temporal discounting model for both gain and loss, both discount functions were closer to the exponential rather than hyperbolic function, indicating more rational (consistent) intertemporal choice. In other words, people make time inconsistent choice because their psychological time has been nonlinearly distorted.

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Table 1. The parameters for temporal discounting model of gain and loss with objective and subjective time.

	Gain				Loss			
	Exponential model	Hyperbolic model	q-exponential model		Exponential model	Hyperbolic model	q-exponential model	
AIC(objective time)	160.74	157.26	136.3884		145.408	143.3939	134.8261	
AIC(Subjective time)	150.76	153.03	136.12		150.25	150.83	132.89	
Parameter	k_e	k_h	k_q	q	k_e	k_h	k_q	q
Objective time	0.0001853	0.000502	0.00825	-6.53676	0.00006355	0.0000882	0.0003701	-5.3188
Subjective time	0.003808	0.0045484	0.00211	3.721846	0.0016431	0.001762	0.0009352	8.20573

Table 2. The parameters for subjective time perception of gain and loss.

	Gain					Loss				
	Linear model	Steven's power law model		Weber-Fechner logarithm model		Linear model	Steven's power law		Weber-Fechner logarithm model	
AIC	84.13048	46.7527		36.61616		80.08146	50.0906		40.20887	
Parameter	a	a	s	a	b	a	a	s	a	b
	0.020726	40.7575	0.1544	15.8245	2.5904	0.018368	23.40066	0.20403	17.05228	0.4454

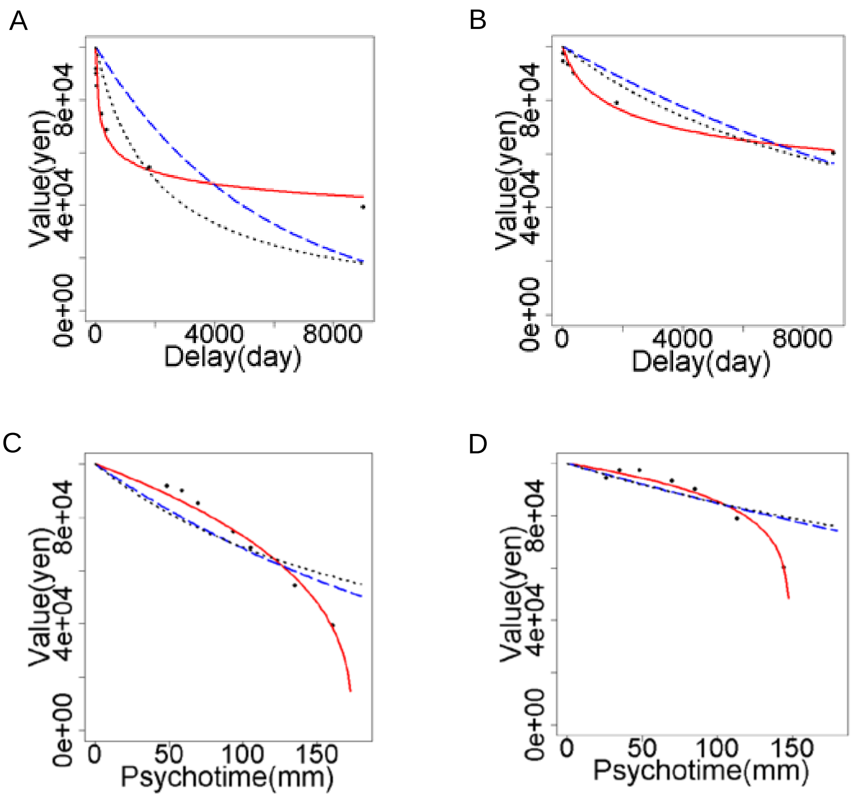

Figure 1. The red solid curve is the q-exponential function. The blue dashed curve is the exponential function. The black dotted curve is the hyperbolic function. (A) Temporal discount functions with delay for gain. The vertical axis shows the subjective value of delayed rewards (100,000yen). The horizontal axis shows physical time delay (days). (B) Temporal discount functions with delay for loss. The vertical axis shows the subjective value of delayed payment (100,000yen). The horizontal axis shows physical time delay (days). (C) Temporal discount functions with psychological time for gain. The vertical axis shows the subjective value of delayed rewards (100,000yen). The horizontal axis shows subjective perception of time delay (mm). (D) Temporal discount functions with psychological time for loss. The vertical axis shows the subjective value of delayed payment (100,000yen). The horizontal axis shows subjective perception of time delay (mm).