

Price Dynamics in a Double Auction Market with Many Securities and Money: A Simulation Study

Michiyoshi Hirota*¹

Abstract

This thesis is devoted to the study of whether or not a traditional double auction trading process in a model with many securities and money can converge to a general equilibrium price. In addition to the traditional presumption of double auction markets we assume in several experiments that artificial subjects with least rationality buy or sell such that their utility never decrease and choose calling prices randomly from a certain possible range which is given by the past history of calling prices and their utility function. This thesis asserts that a double auction process under this assumption tends to approach a general equilibrium price, while the distribution of final allocations may be somewhat dispersed.

Keyword: double auction, computer simulation, equilibrium price

JEL Classification Numbers: C61, C63

1. Introduction

Since Smith (1962) proposed an experimental concept of double auction, many experiments about double auction market have been done, and those results well tell us that Walras central market can be analyzed deeply through examining decentralized markets such as double auction. In the first stage, the study of double auction has been restricted to one commodity model which is described by demand and supply curves. In that framework, a sequence of transaction prices is seen to move to an equilibrium, guaranteeing some efficiency. Now it is not so difficult to see why such a convergent phenomenon may be guaranteed in a special framework of double auction. Almost all experiments have been done by using human subjects, which gradually interests us in a study of human psychological matter and the assumption of rationality of economic theory.

In this paper we want to show through several computer experiments that a double auction trading process in our model with many securities and money tends to converge to a general equilibrium price.

2. Basic model with many securities and money

In this section we want to consider a basic model used for our simulation study of a double auction process on price formation: Our basic pure trade model involves a certain kinds of securities and money, and many types of individuals each of whom has his own dividend rate vector for possible states and a given probability for each state which is assumed to be common among all individuals. Our final purpose of this paper is to study whether or not some decentralized pricing process such as

*¹ Graduate school of economics, Aoyama Gakuin University. E-mail 1:michi.hirota@nifty.com 2:d2109001@aoyama.jp

double auction will converge to a general equilibrium price and will be somewhat related to a Walrasian tatonnement process.

Let us proceed to the explanation of our model. The number of goods is assumed to be $n+1$, and from the first to the n -th good denotes securities and the $(n+1)$ -th money. Let an initial holding of the j -th individual be given by

$$\omega^j = (\underbrace{\omega_{1j}, \dots, \omega_{nj}}_{\text{securities}}, \underbrace{\omega_{n+1,j}}_{\text{money}}), \quad j = 1, 2, \dots$$

Assumption 1 (dividend rate): Let the number of states be m . We assumed that the i th security's dividend rate for the j -th individual is given or anticipated as

$$d_{ij} = (d_{ij}^1, \dots, d_{ij}^m), \quad j = 1, 2, \dots, i = 1, 2, \dots, n$$

When state k occurred, the total dividend for the j -th person would become

$$\sum_{i=1}^n d_{ij}^k \hat{x}_{ij} + \hat{x}_{n+1,j} \quad (2-1)$$

However each state is supposed to happen in probabilistic manner, so that the total expected dividend will be represented as

$$\sum_{k=1}^m q_k \left(\sum_{i=1}^n d_{ij}^k \hat{x}_{ij} \right) + \hat{x}_{n+1,j}, \quad (2-2)$$

where $q = (q_1, q_2, \dots, q_m)$ denotes the probability vector of all the states.

Assumption 2 (Utility function): It is assumed that utility function $U(W)$ is concave in a certain range of wealth W , i.e.,

$$U'(W) > 0 \quad \text{and} \quad U''(W) \leq 0 \quad . \quad (2-3)$$

For a portfolio $(x_{1j}, \dots, x_{nj}, x_{n+1,j})$ we have the following expected utility:

$$F(x_{1j}, \dots, x_{nj}, x_{n+1,j}) = \sum_k q_k U \left(\sum_{i=1}^n d_{ij}^k x_{ij} + x_{n+1,j} \right) .$$

When the hypothesis of expected utility is prevalent, the problem of choosing an optimal portfolio can be reduced to solving the following concave programming:

$$(\Delta - 1) : \quad \max \quad F(x) = \sum_k q_k U \left(\sum_{i=1}^n d_{ij}^k x_i + x_{n+1} \right) \quad (2-4)$$

$$\text{subject to} \quad \sum_{i=1}^{n+1} p_i x_i \leq M \quad (2-5)$$

where $p = (p_1, \dots, p_{n+1})$ is the price vector and M is the nominal income. In the above the individual's number j will be omitted in order to make the notation more simple.

The solution for $(\Delta - 1)$ is given as a saddle point of Lagrange function the condition of which is considered to be the Kuhn-Tucker condition as follows:

$$\frac{\partial F(\cdot)}{\partial x_i} - \lambda p_i \leq 0 \quad (2-6)$$

$$x_i \left(\frac{\partial F(\cdot)}{\partial x_i} - \lambda p_i \right) = 0 \quad (2-7)$$

Let the demand functions obtained as the solution for $(\Delta - 1)$ be denoted by

$$x_i = f_i(p_1, \dots, p_{n+1}, M) \quad (2-8)$$

Summing up the above demand functions for all the individuals, we have the market excess demand functions:

$$E_i(p_1, \dots, p_{n+1}) = \sum_j f_{ij}(p_1, \dots, p_{n+1}, M_j) - \sum_j \omega_{ij} \quad , i = 1, \dots, n+1 \quad (2-9)$$

where $M_j \equiv \sum p_i \omega_{ij}$. The market excess demand functions $E_i(p)$'s satisfy: (i) $E_i(p)$'s are continuous on a unit simplex $S^n = \{p \mid \sum_{i=1}^{n+1} p_i = 1, p_i \geq 0\}$, (ii) $E_i(p)$'s are homogeneous of degree zero, and (iii) $E_i(p)$'s satisfy the Walras law $\sum_{i=1}^{n+1} p_i E_i(p) \equiv 0$.

Definition (Market equilibrium): A price vector $p^* = (p_1^*, \dots, p_{n+1}^*)$ up to multiplicity is called a general equilibrium price vector if $E_i(p_1^*, \dots, p_{n+1}^*) = 0$ holds for all i .

3. Two examples of utility function and computation of equilibrium

In this section we want to propose a model with the specific forms of utility function which have been used in our computer experiments. The quadratic function we adopt is given by

$$U(W) = W - \frac{1}{2b} W^2 \quad (3-1)$$

where W denotes a nominal income and b denotes a parameter that determines a saturation point of utility. This utility function (3-1) expresses special case implying patterns of risk aversion. So far we have two theoretical methods for analyzing the problem of portfolio selection, that is, (i) the method of maximizing the expected utility over some constraint, which was proposed by Von Neumann, and (ii) the method of maximizing the choice function with average return and its variance of portfolio, which was proposed by Markowitz (1968). Whether or not the hypothesis of expected utility may be correct in terms of actual human behavior has been intensively discussed by Kahneman and Tversky (1979). However some results of experiments (for example, Plott and Sunder (1988)) seem to support this hypothesis.

In our computer experiments we suppose this hypothesis. The expected utility constructed by this specific utility (3-1) leads to, as its result, a choice function two elements of which depend only on average return and variance of portfolio. This means that the above two methods become consistent.

[Risk aversion case]

We want to use the following notation in which individual number is omitted for the simplicity:

$$\text{probability distribution} : q = (q_1, q_2, \dots, q_m)$$

$$\text{dividend rate matrix} : d = \begin{bmatrix} d_1^1 & d_1^2 & \dots & d_1^m \\ d_2^1 & d_2^2 & \dots & d_2^m \end{bmatrix},$$

where q_i denotes the probability of state i and d_r^k denotes the dividend rate of security r that is given him when state k occurs.

Let us define the expected average of each one unit security and the variance-covariance matrix of two securities as

$$\text{average} : (a_1, a_2) = \left(\sum_k q_k d_1^k, \sum_k q_k d_2^k \right)$$

$$\text{variance - covariance matrix} : V = \begin{bmatrix} \sum_k q_k (d_1^k - a_1)(d_1^k - a_1) & \sum_k q_k (d_1^k - a_1)(d_2^k - a_2) \\ \sum_k q_k (d_2^k - a_2)(d_1^k - a_1) & \sum_k q_k (d_2^k - a_2)(d_2^k - a_2) \end{bmatrix}.$$

Then we have the average return and the variance for a portfolio $x = (x_1, x_2, x_3)$ as follows;

$$\text{average return} : a(x) = a_1 x_1 + a_2 x_2 + x_3$$

$$\text{variance} : (c(x))^2 = \begin{bmatrix} x_1 & x_2 \end{bmatrix} V \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

When utility function (3-1) is assumed, simple calculation yields the following expected utility for a portfolio $x = (x_1, x_2, x_3)$;

$$F(x_1, x_2, x_3) = a(x) - \frac{1}{2b} [(a(x))^2 + (c(x))^2]. \quad (3-2)$$

Since we suppose the hypothesis of expected utility, each individual obtains the demand for security and money by maximizing the expected utility subject to the budget constraint

$$\begin{aligned} (\Delta - 1) : \max & \quad F(x_1, x_2, x_3) \\ \text{subject to} & \quad \sum_{i=1}^2 p_i x_i + x_3 \leq M \equiv \sum_{i=1}^2 p_i \omega_i + \omega_3 \end{aligned}$$

where p_i denotes the price of security i and ω denotes the initial endowment of this person.

To get the solution of $(\Delta - 1)$ by an ordinary direct way of using the marginal conditions seems to become complicated and tedious. Therefore we want to adopt the method of two steps.*²

Example: Let us consider an economy in which there are two types of security, three states with the probability $q = (0.30, 0.25, 0.45)$, and two types of individuals.

As to the first individual:

$$\text{dividend rate matrix} \quad d_1 = \begin{bmatrix} d_{11}^1 & d_{11}^2 & d_{11}^3 \\ d_{21}^1 & d_{21}^2 & d_{21}^3 \end{bmatrix} = \begin{bmatrix} 60 & 50 & 50 \\ 32 & 35 & 20 \end{bmatrix}$$

*² We want to omit details of this method from this preprint.

initial endowment vector $\omega^1 = (\omega_{11}, \omega_{21}, \omega_{31}) = (9, 7, 250)$

saturation point $b = 1000.$

As to the second individual:

dividend rate matrix $d_2 = \begin{bmatrix} d_{12}^1 & d_{12}^2 & d_{12}^3 \\ d_{22}^1 & d_{22}^2 & d_{22}^3 \end{bmatrix} = \begin{bmatrix} 67 & 60 & 60 \\ 37 & 41 & 26 \end{bmatrix}$

initial endowment vector $\omega^2 = (\omega_{12}, \omega_{22}, \omega_{32}) = (10, 14, 200)$

saturation point $b = 1200.$

When we try to use method of fixed point algorithm, we will have the following equilibrium vector and final endowments at equilibrium:

equilibrium vector $p^* = (p_1^*, p_2^*) = (27.3459, 18.1856)$

final endowments $\omega^1 = (9.9676, 6.24019, 211.094)$ $\omega^2 = (9.0324, 14.7598, 238.906)$.

4. Experimental Results

4.1. Experiments in non-linear utility model

In Section 3 we have studied an example of parameters in non-linear utility case, and have computed the general equilibrium price in each example by using three methods of algorithm and calculation by hand. This is the story of the traditional Walrasian world in which central market with auctioneer will be somewhat artificially presumed. My concern in this paper is to clarify whether or not our trading process based on the rule of double auction can well bring about some trade prices which are close to a Walrasian general equilibrium price.

Let us show one of the results in our experimental works by the computer simulation.

Result: Let us consider an economy with the parameters of Example in Section 3. In addition, we assume that the number of each type agent is thirty, so the total number of agents is sixty, and minimum trading price is one and maximum trading price is the largest amount of money which participants initially hold, and the amount of security in one time trade is unity.

Figure 4-1 and Figure 4-2 show the distributions of trade prices of the first security and the second security with trade count.

As these figures show, at the beginning of trading in market, actual trade prices look much dispersed, but the phenomenon of this dispersion gradually looks disappeared according as trading proceeds, and at the ending of trading in market, a sequence of actual trade prices seems to enter in very small neighborhood of the Walrasian equilibrium prices. This enables us to say that our trading process in decentralized market well produces, as actual transacted prices, an equilibrium price.

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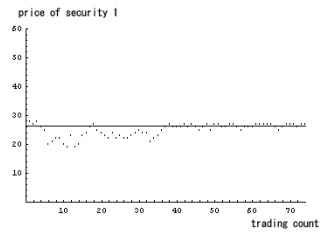


Figure 4-1 (distributions of trade prices of the first security)

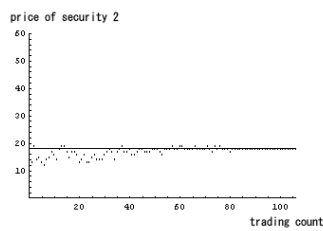


Figure 4-2 (distributions of trade prices of the second security)

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