On Some Bilateral Exchange Method and Its Dynamics

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Abstract

It is very important study of investigating a relation between an abstract Walras central market and possible decentralized markets which will be behind abstract central market. Recently Gintis (2007) has studied the price dynamics in a bilateral decentralized exchange economy based on an agent based simulation and private prices of agents, in which Scarf model is also adopted. In his paper Gintis asserts that a sequence of distribution of private prices and a sequence of the average of private prices will converge to the general equilibrium, while both a tatonnemnet and a double auction trading process show instability. If his assertion turns out to be true irrespectively of choice of preferences and initial holdings, his method seems to take a position in the study of finding a general equilibrium price. In this paper we try to follow a method of Gintis and to do several simulation experiments by our own computer program, and to examine whether or not his assertion can be true as a general tendency.

Keyword：
Bilateral exchange economy, computer simulation, equilibrium price

1. Introduction

The problem of discovering a general equilibrium in Walras central market was intensively in the 1950s studied by Arrow and Hurwicz (1958), Arrow and Block and Hurwicz (1959) and others: The most important result is that when we put a fairly strong condition as "gross substitutes" on the market excess demands, a Walrasian price adjustment process (called tatonnement) is globally stable. Whether or not a tatonnement can be stable without this assumption was left unclear. Also in Arrow and Hurwicz on conjecture had been proposed; a tatonnement process may be possible to be stable without this condition if we well utilize the property of convexity of preferences.

Soon Scarf (1960) gave a negative answer to the above conjecture by showing two counter examples; one is concerned with Leontief type utility function with no substitutability, and another is concerned with C.E.S. type utility function. Later this demonstration seems to be

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reconfirmed by a series of the study of market excess demand functions which was initiated by Sonnenschein (1972) and further developed by Debreu (1974) and others. Since then, theorists have changed a direction of study about the discovery of general equilibrium from Walrasian tatonnement to computation of solving a fixed point (See Scarf (1972)).

However it seems that there still remains an important study of investigating a relation between an abstract Walras central market and possible decentralized markets which will be behind abstract central market. The double auction market proposed by Smith (1965) and still now continuing to be developed by Plott and many others will be a candidate to decentralized markets. By a series of researches of this market the relation between general equilibrium and decentralized markets like double auction market is now becoming clear. In particular, Anderson, Plott, Simomura and Granat (2004) suggest us that the path of the average of transaction prices in double auction market looks like following a Walrasian price adjustment process: In their experiments they adopt the model Scarf proposed as an example of instability of tatonnement, and when a model predicts a limit cycle the data in experiment also depicts a kind of limit cycle, and when a model predicts a convergence of equilibrium the data shows similar convergence.

Recently Gintis (2007) has studied the price dynamics in a bilateral decentralized exchange economy based on an agent based simulation and private prices of agents, in which Scarf model is also adopted. In his paper Gintis asserts that a sequence of distribution of private prices and a sequence of the average of private prices will converge to the general equilibrium, while both a tatonnemnet and a double auction trading process show instability. If his assertion turns out to be true irrespectively of choice of preferences and initial holdings, his method seems to take a position in the study of finding a general equilibrium price. In this paper we try to follow a method of Gintis and to do several simulation experiments by our own computer program, and to examine whether or not his assertion can be true as a general tendency.

2. Bilateral Exchange Method With Two Goods

In his paper Gintis actually discuss two models; (i) Scarf type model of exchange, and (ii) Walrasian production economy. However, in order to clarify whether or not this Gintis' method can be true as a general tendency irrespective of the choice of utility functions and initial endowments, in this paper we want to concentrate on the problem about only exchange model which seems to be fundamental to Gintis' method. In this section we study two goods exchange economy with Gintis' bilateral method, which will show us an intrinsic nature of his method, though only the three goods exchange model was handled in the Gintis' paper.

Let us consider an exchange economy with two types of individuals and two goods and a situation in which there exist n agents of each type, so that totally 2n. For convenience, we number all agents from 1 to 2n, for example, agents of type 1 from 1 to n, agents of type 2 from n+1 to 2n.

Next we proceed to the problem of how agents trade each other, and to describe a Gintis' way of exchange in the starting period, say, period 1.

[trade procedure in period 1]

(i) we take the second good as *numeraire*, so that the price of second good is unity. Let us give all agents private prices by the following way; take supremum and infimum price, say, hp and lp, and consider an interval [lp, hp] and take a set S of lattice points in the above interval. Suppose a private price vector of each agent is randomly chosen from S, say, *{p(1), p(2),…, p(2n)}*, which may be considered to be fairly uniformly distributed in S if n is fairly large.

(ii) All agents can be sometimes "initiator" or "responder". Suppose agent *i* is an initiator and agent *j* is responder. In this case agent *i* only can offer a trade term, that is, an exchange of some amount of good *k* for amount of good *r*. On the other hand, agent *j* only determines whether to accept or reject this trade term offered by agent *i*.

Next we have to think about how two agents come across, in a market, as initiator or responder. For this matching problem, we want to take a simple way such that an initiator is, in turn from the agents set {1, 2,…, 2n}, chosen, and a responder is randomly chosen from the set in which this initiator is excluded. In our experiment we will show later, all agents are assumed to be initiator one time in the simulation in this section and *r* times in the next section. (In Gintis(2007), r=1 is assumed.)

Next let us consider the problem of agent's behavior patterns.

[behavior pattern of initiator]

Agent *i* determines net demand or net supply by maximizing the utility function subject to the budget constraint with his own private price.

[behavior pattern of responder]

The behavior pattern of responder is very simple: Suppose agent *i* (initiator) give a specific trade term to agent *j* (responder). If this exchange brings about the non-decrease of the value evaluated by agent *j*'s private price, he will be willing to accept this offer, and reject otherwise.

Next we proceed to the problem of trading in subsequent periods.

Assumption 1 (Renewal of initial holdings): After trades in a period, the final stock of goods is assumed to be consumed by agents. The same initial holdings as in period 1 are again given agents before starting trade in next period.

It is assumed that the private prices *{p(1), p(2),…, p(2n)}* given in the starting period continue to be kept until *t* times of periods are finished. And we call *t* times of periods one generation. Of course, the final stock of agents may change from period to period.

After on generation is over, all agents can calculate the total of utility value over period, and compare those to each other. For example, take agent *i* and agent *j* of type 1, and suppose that a sequence of their final stock in period is

$$\{x(i,1), x(i,2),\ldots, x(i,t)\} \text{ and } \{x(j,1), x(j,2),\ldots, x(j,t)\}.$$

When estimating the above by their common utility function and summing up, we put the following rule.

Imitation rule: If total utility of agent *i* > total utility of agent *j*, then agent *j* tries to imitate private price *p(i)* and vice varsa.

In addition to the imitation of private prices, we finally put the mutate assumption on change of prices.

Assumption 2 (Mutation): r percentage of all agents mutate, and make their private prices increase or decrease change by a certain percentage.

The Gintis' assertion is as follows.

Gintis's assertion: According as generation proceeds, a possible distribution of private prices that can be obtained at each generation will gradually change from the uniform distribution at the starting period, and the average of distributions at each generation will converge to the general equilibrium price.

Our concern is to clarify whether or not this assertion will be true. We will show that in final submission paper in detail.

A bilateral exchange method with agent based simulation proposed by Gintis may be stimulating for the further researches, in particular, (i) the relationship between tatonnement and non-tatonnement process and various decentralized markets such as double auction markets, and (ii) the relationship between Gintis type bilateral exchange method and other type bilateral exchange (for example, see Foley (1994)) and etc.

However, unfortunately this Gintis' method does not necessarily guarantee to discover equilibrium price contrary to Gintis' assertion, what are the main sources to bring about non-convergence phenomenon?

At the starting period the private prices of all agents may be considered to be fairly uniformly distributed, and according to the proceed of periods, some of private price will disappear gradually by the imitation process we suppose, and if some private price that is given initially disappeared through trading process, such a price would never appear again. Therefore if some price very close to the equilibrium price disappears, such a price can not be restored to possible agent price since then. This may be due to the fact that Gintis method does not take a new creation process of prices into consideration. In order to improve this defection and obtain the stable function of bilateral exchange market, we have to adopt a fairly new method which will be different from Gintis.

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